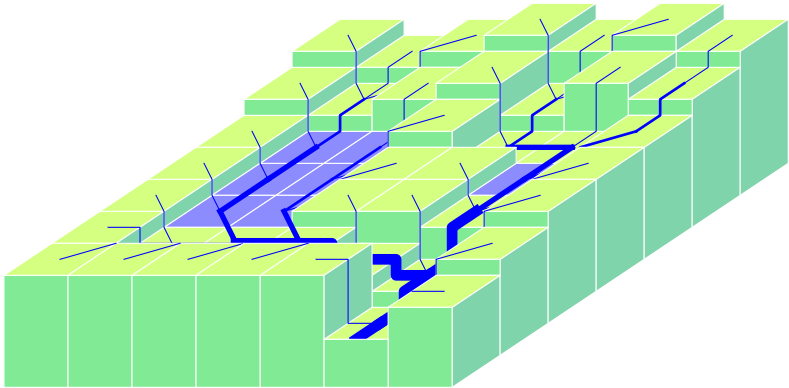
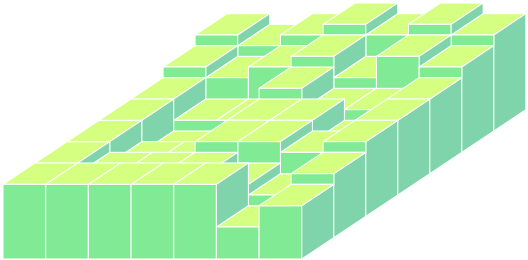
Simple I/O-efficient flow accumulation on grid terrains



Herman Haverkort
TU Eindhoven

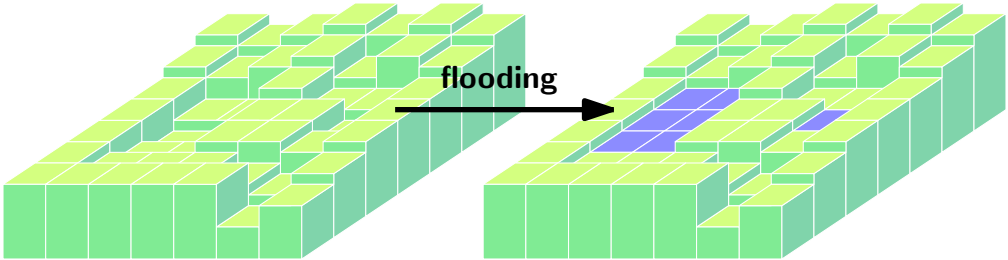
Jeffrey Janssen
Realworld Systems

Drainage network analysis



grid elevation model

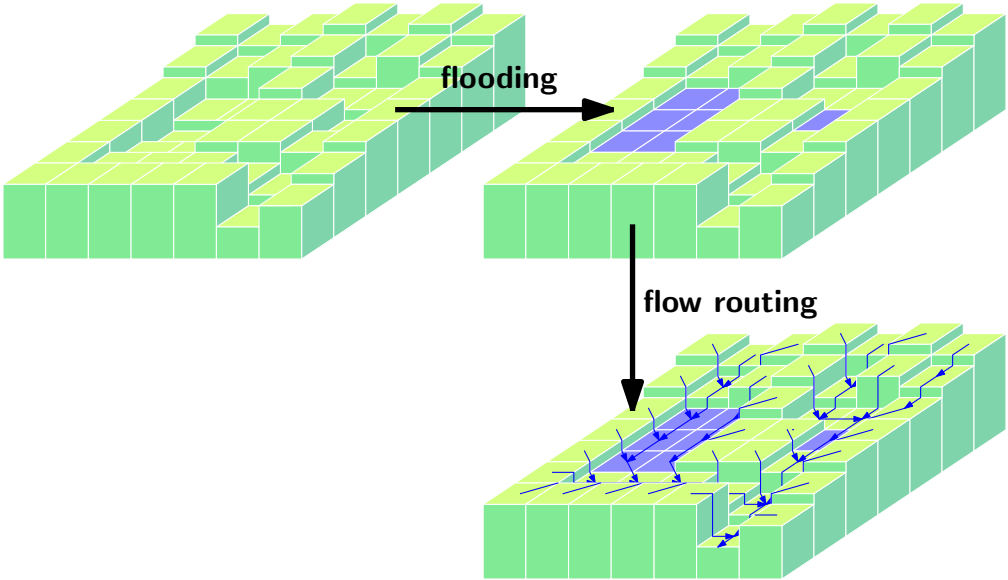
Drainage network analysis



grid elevation model

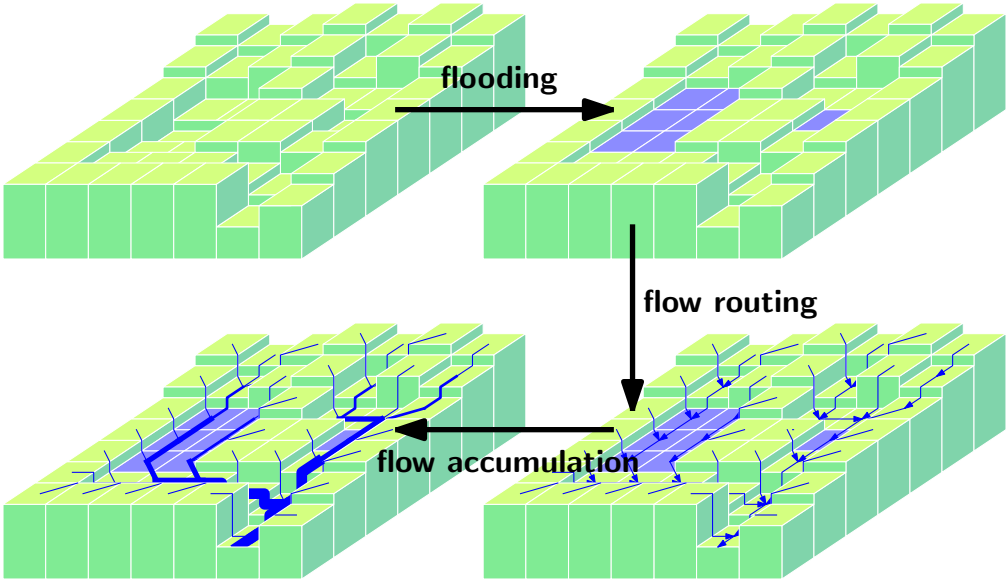
each cell has non-ascending path to boundary

Drainage network analysis



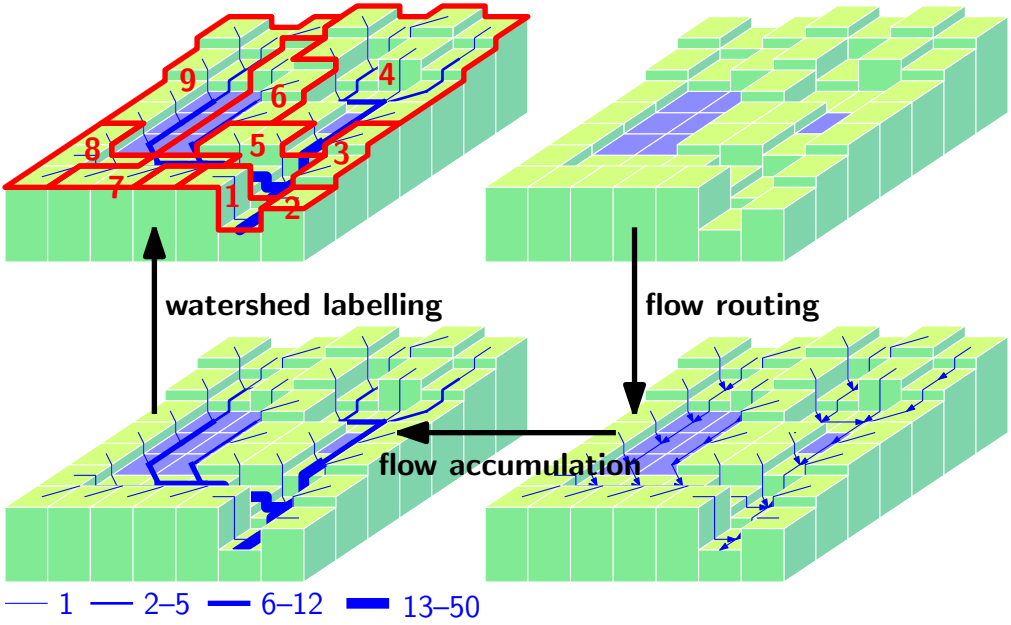
each cell has a flow direction; following flow directions leads to boundary

Drainage network analysis

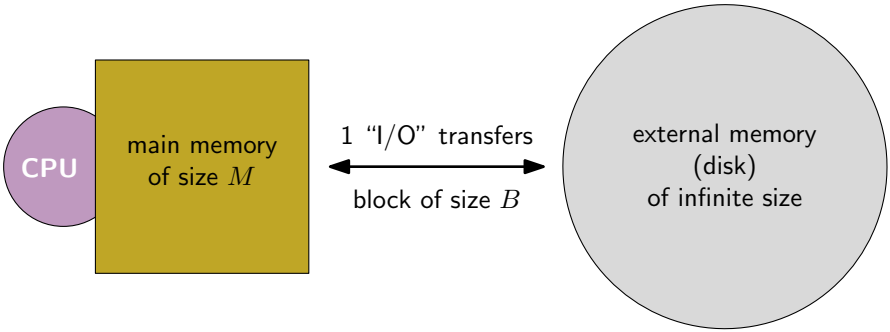


compute for each cell c, from how many cells water passes through c

Drainage network analysis



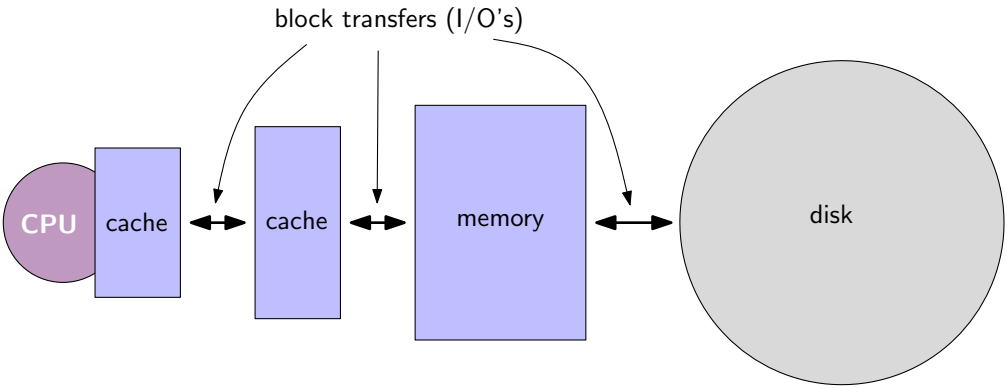
Analysing I/O-efficiency



CPU only operates on data in main memory (for free)

I/O-efficiency = number of I/O's as function of M, B, and grid size N
(sometimes assume $M \geq c \cdot B^2$)

Analysing I/O-efficiency



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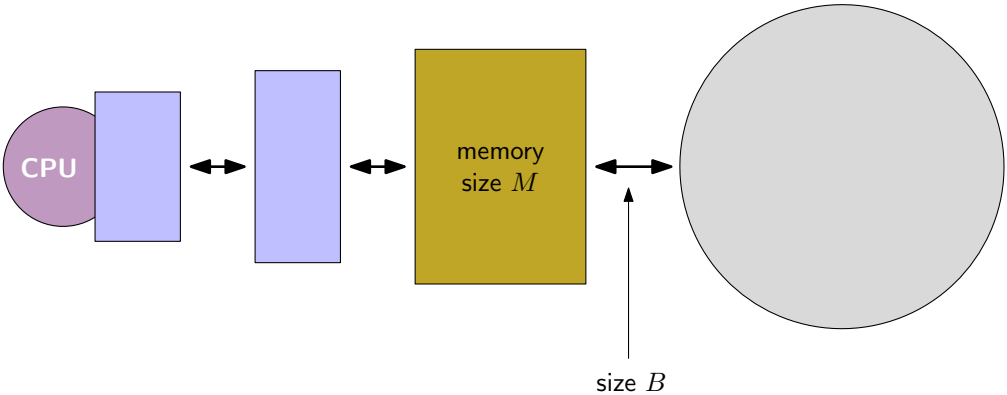

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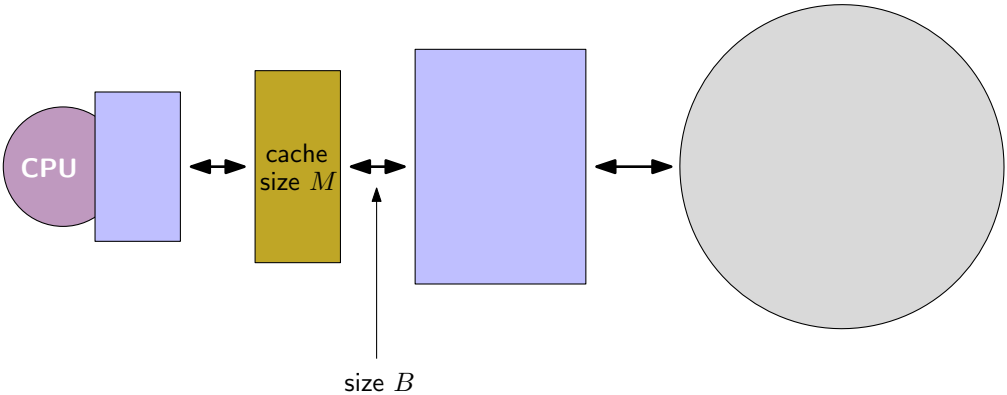

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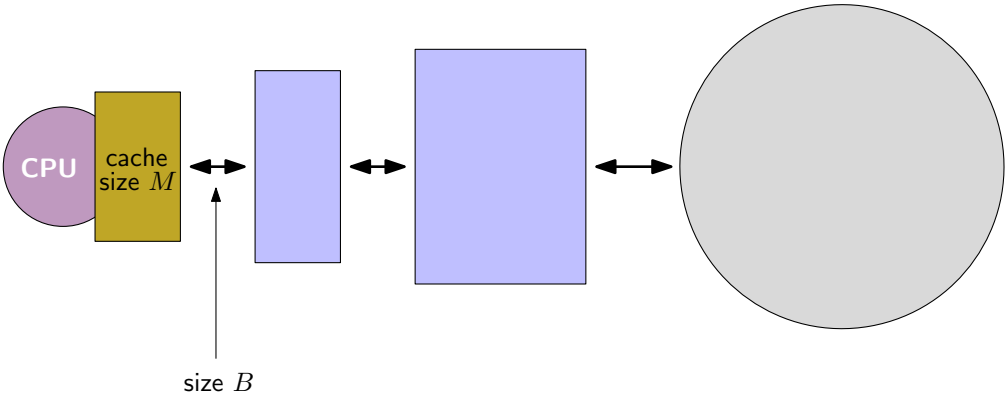

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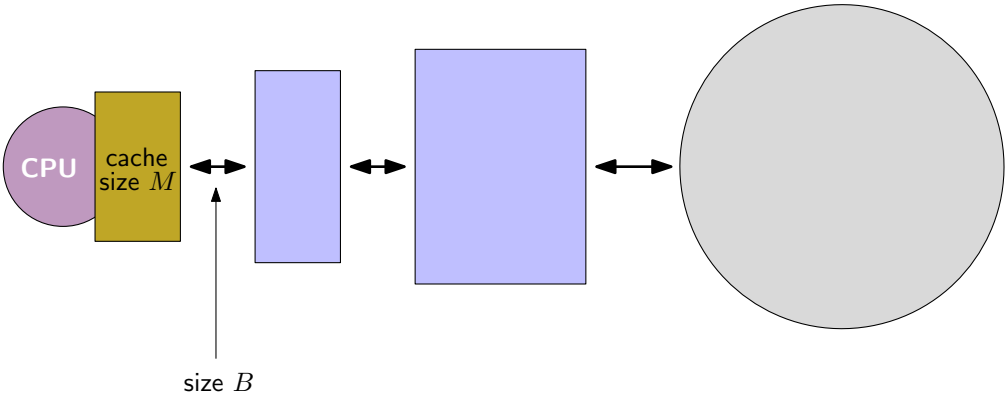
Analysing I/O-efficiency

cache-aware algorithms: know M and B, control caching
cache-oblivious algorithms: do not know M and B, caching left to system



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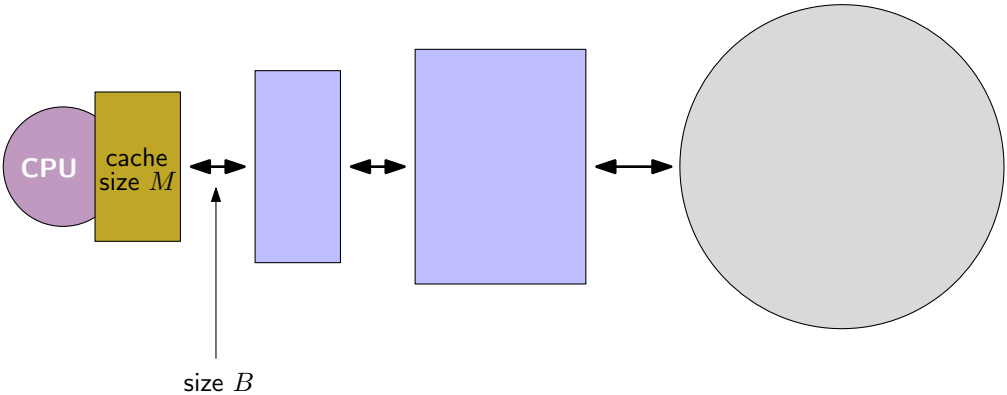
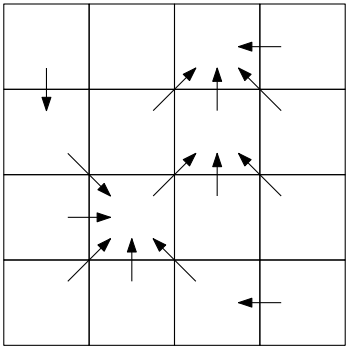


CPU only operates on data in main memory (for free)

scanning N cells takes $\Theta(\mathrm{scan}(N)) = \Theta(N/B)$ I/O's
sorting N cells takes $\Theta(\mathrm{sort}(N)) = \Theta(\frac{N}{B} \log_{M/B} \frac{N}{B})$ I/O's

Time-forward processing (Arge et al.)

Goal: compute flow accumulation for each cell c
= #cells from which water passes through c
= size of tree rooted at c



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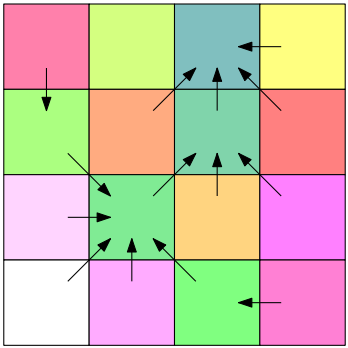

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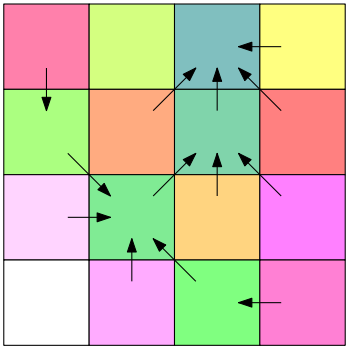
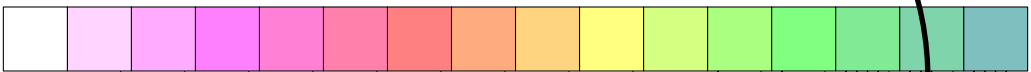


sort

output



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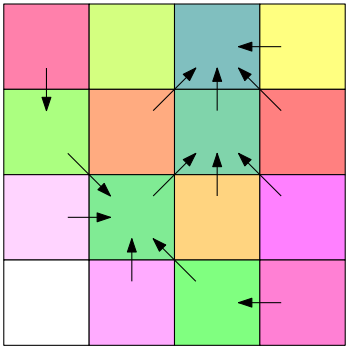
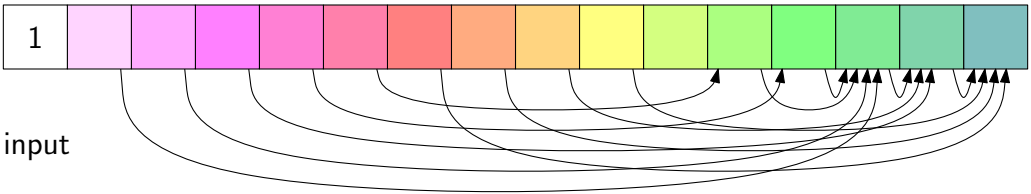


output



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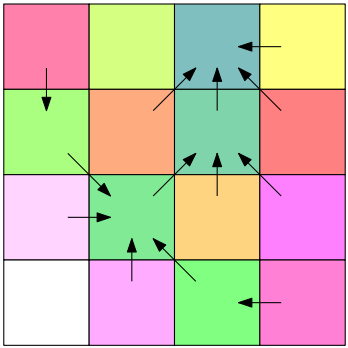

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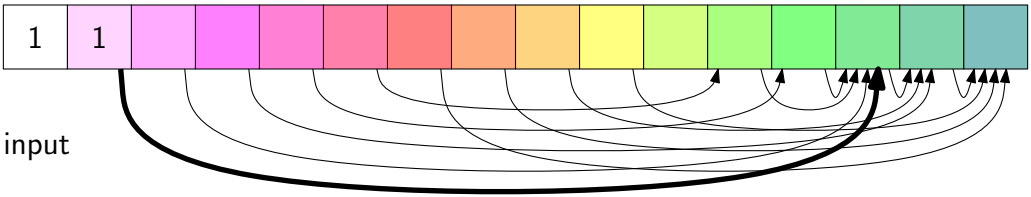

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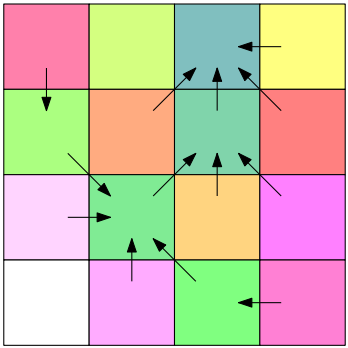

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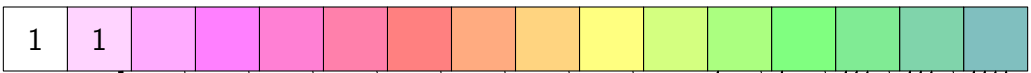

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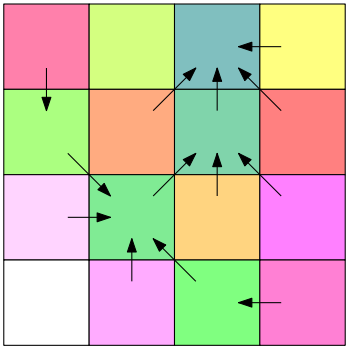
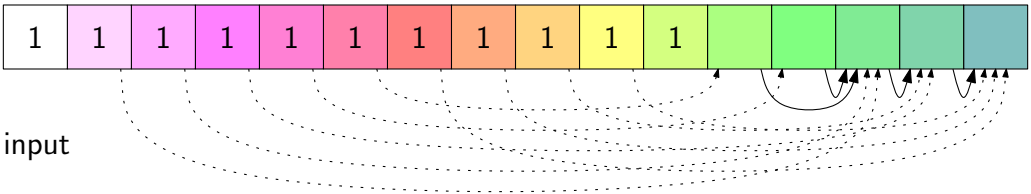


output



input

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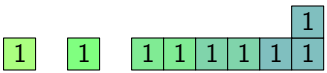

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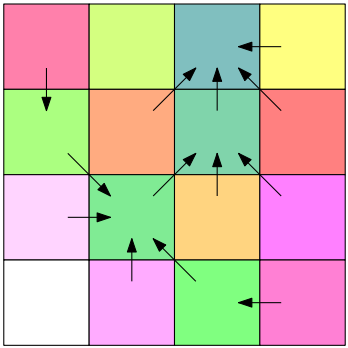

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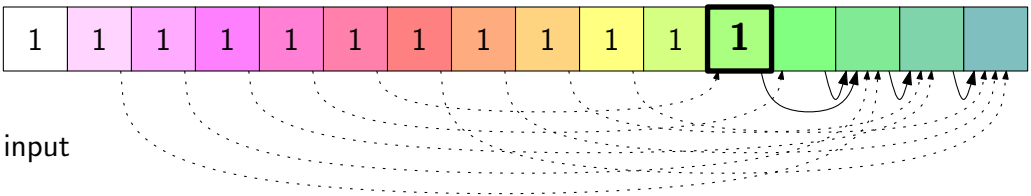

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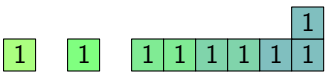

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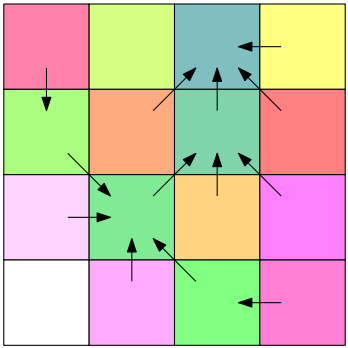

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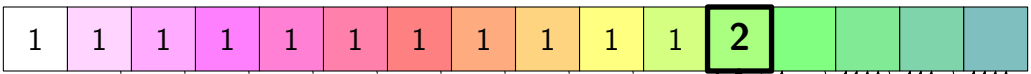

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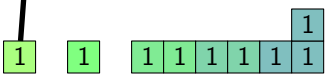

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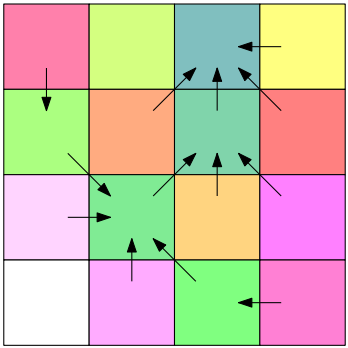

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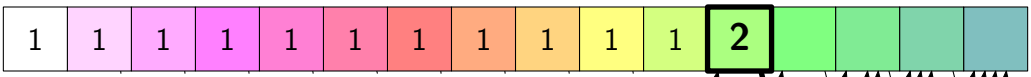

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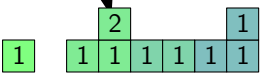

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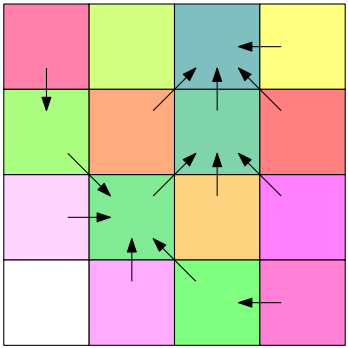

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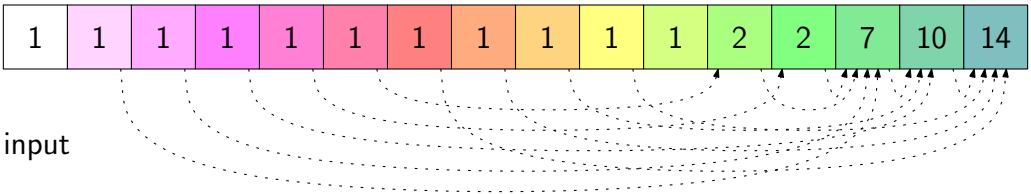

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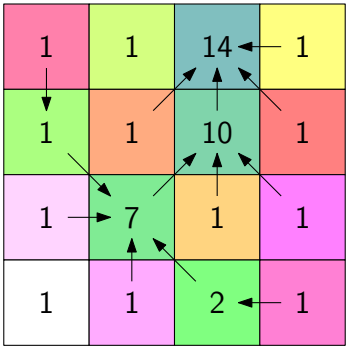
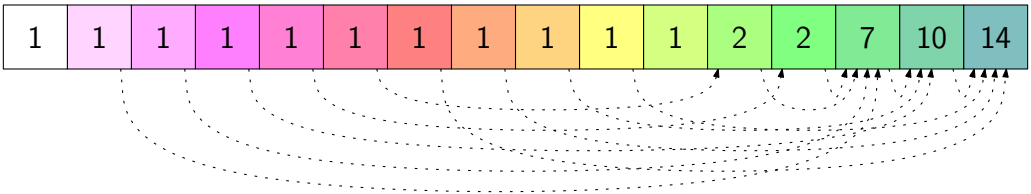


sort

output



Time-forward processing

Worst-case I/O's: $\Theta(\mathrm{sort}(N))$ (Arge et al.)

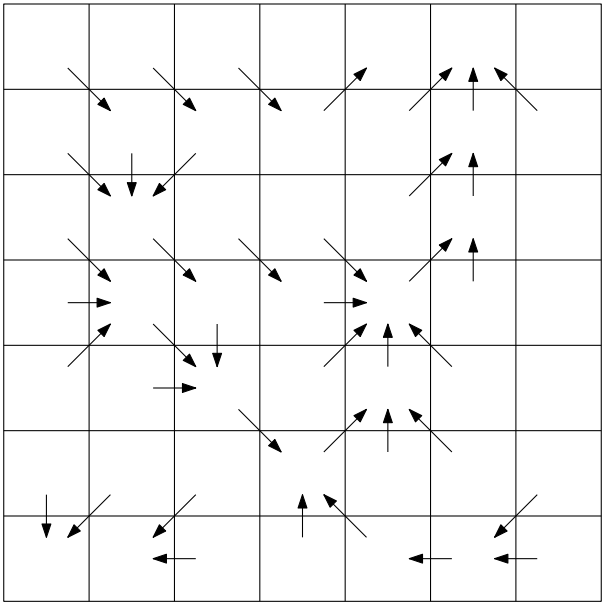
I/O-volume per grid cell (optimistic):

Sorting grid into list of (xy-location, topological nr., out-neighbour top. nr.)	$2 \times 2 \times 24 = 96$ bytes
Flow accumulation, input: Flow accumulation, output: (xy-location, flow)	24 bytes 16 bytes
Sorting output into grid	$2 \times 2 \times 16 = 64$ bytes
Total:	**200 bytes**

I/O-volume per grid cell (pessimistic):

Sorting grid into list of (xy-location, topological nr., out-neighbour top. nr.)	$3 \times 2 \times 24 = 144$ bytes
Flow accumulation, input: Flow accumulation, priority queue: Flow accumulation, output: (xy-location, flow)	24 bytes $2 \times 16 = 32$ bytes 16 bytes
Sorting output into grid	$3 \times 2 \times 16 = 96$ bytes
Total:	**312 bytes**

Naïve algorithm



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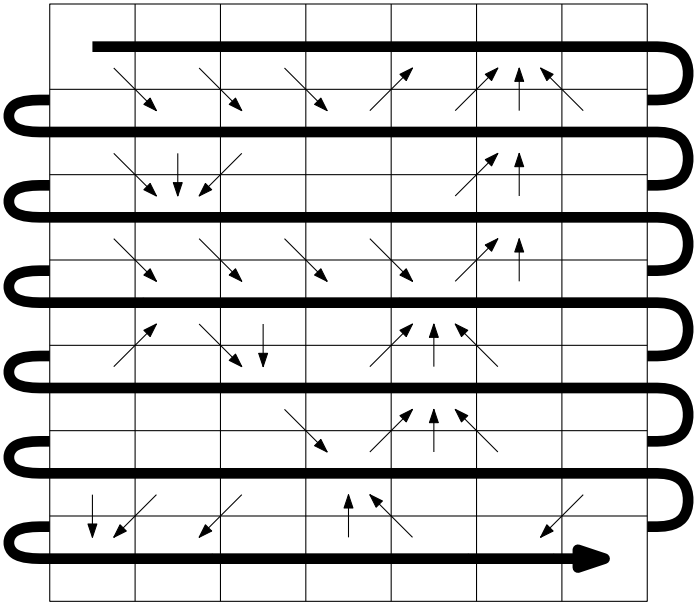

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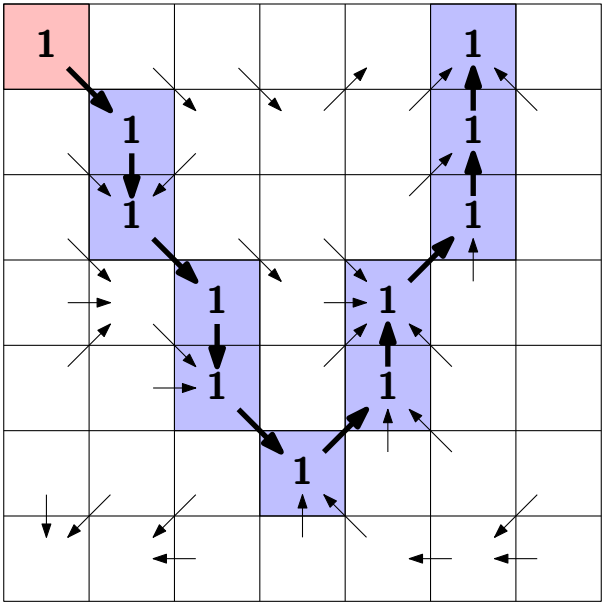

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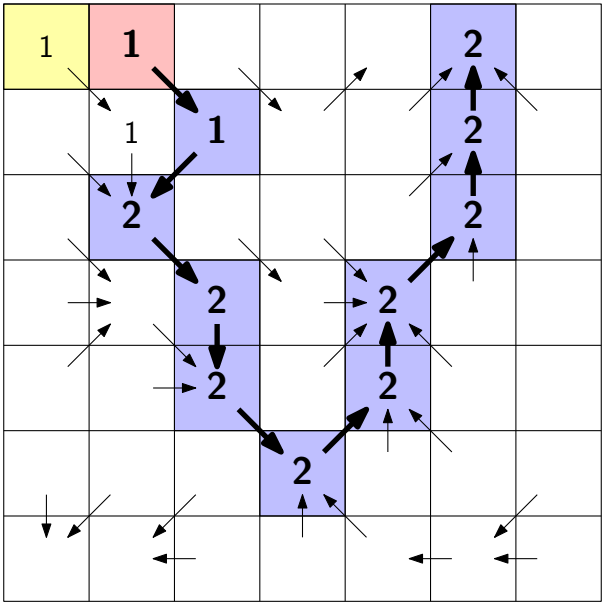

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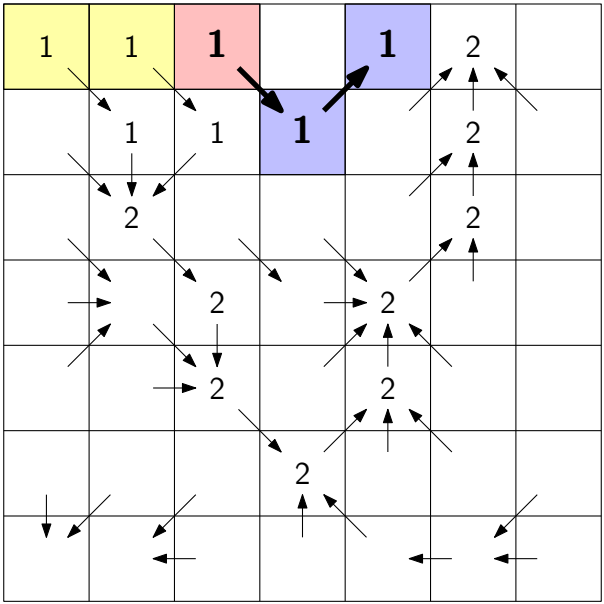

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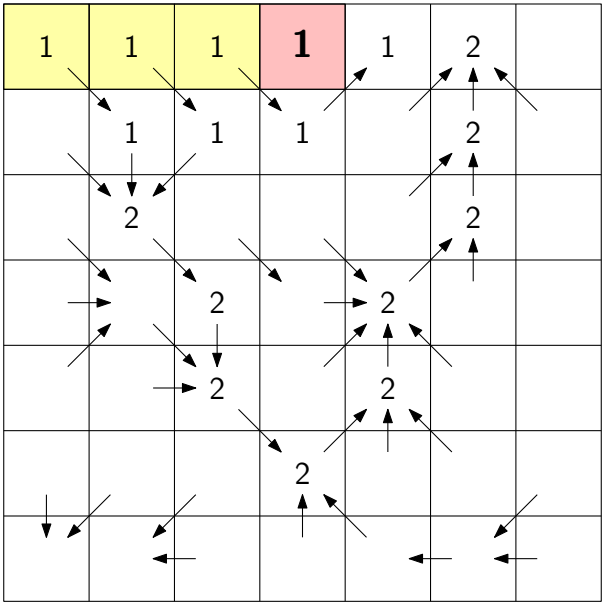

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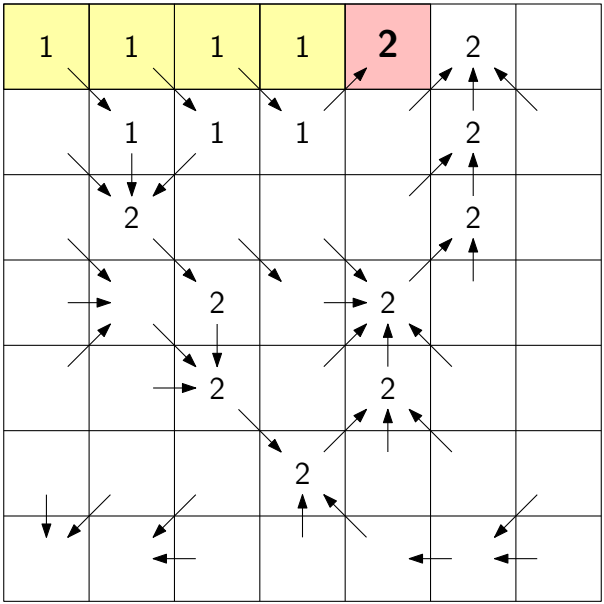

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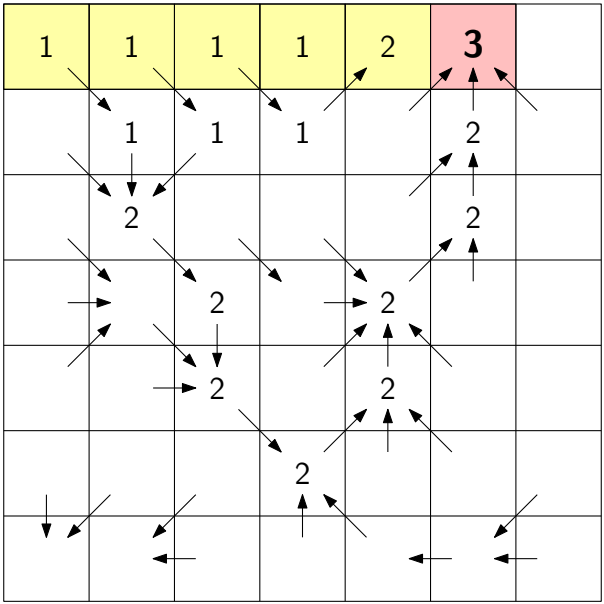

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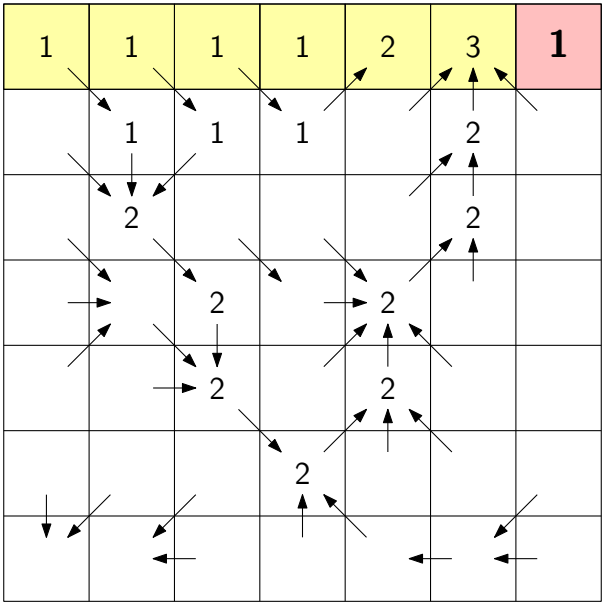

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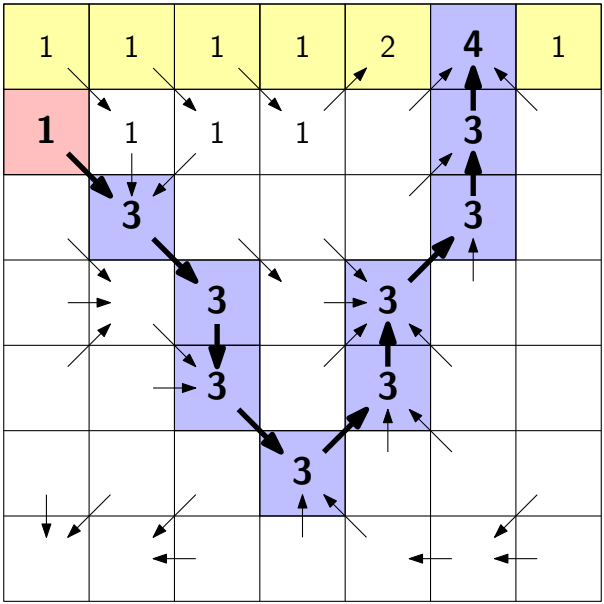

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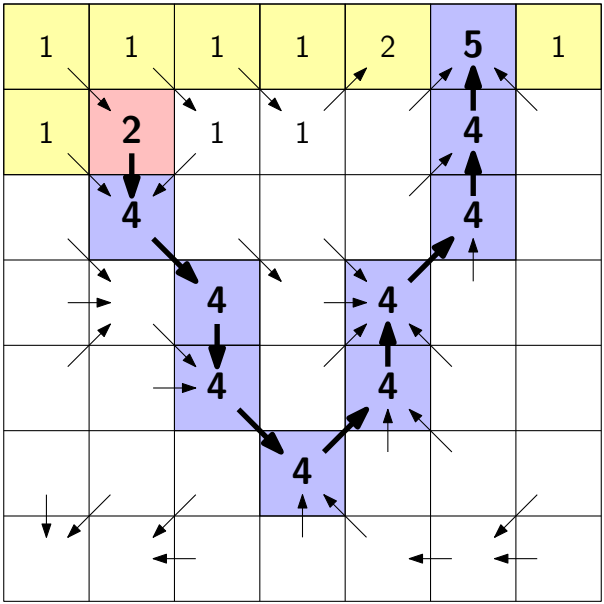

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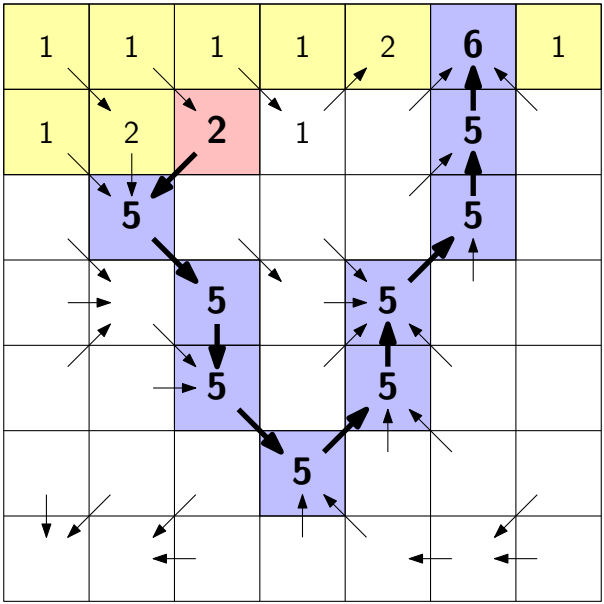
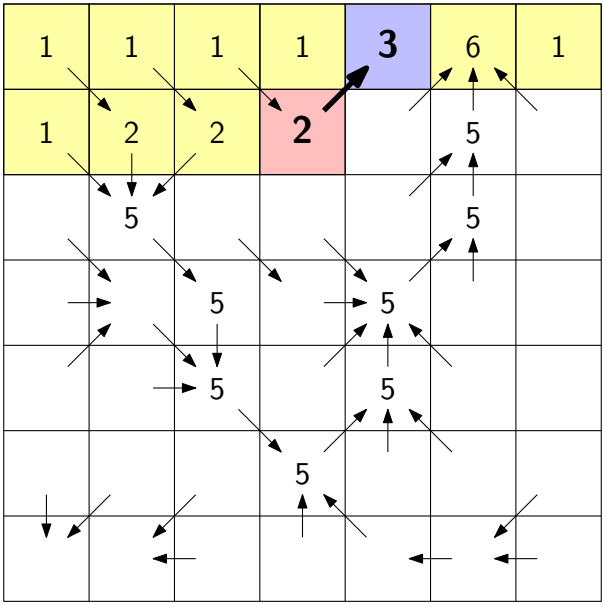
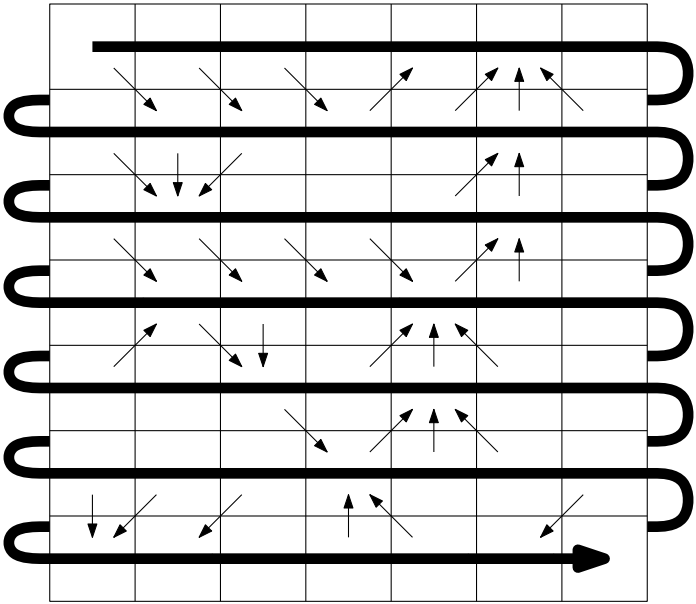


Naïve algorithm

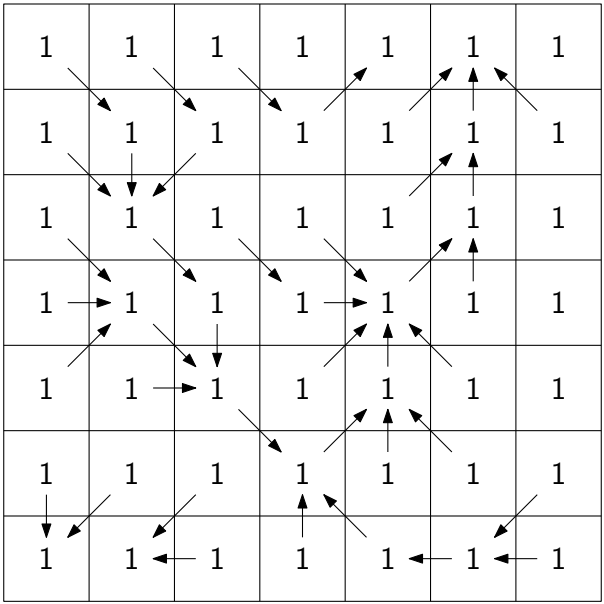


Worst-case running time: $\Theta(n^2)$

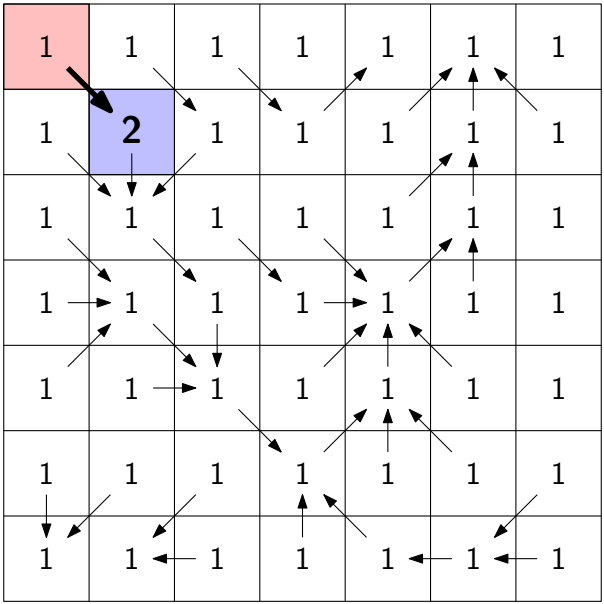
Row-by-row scan



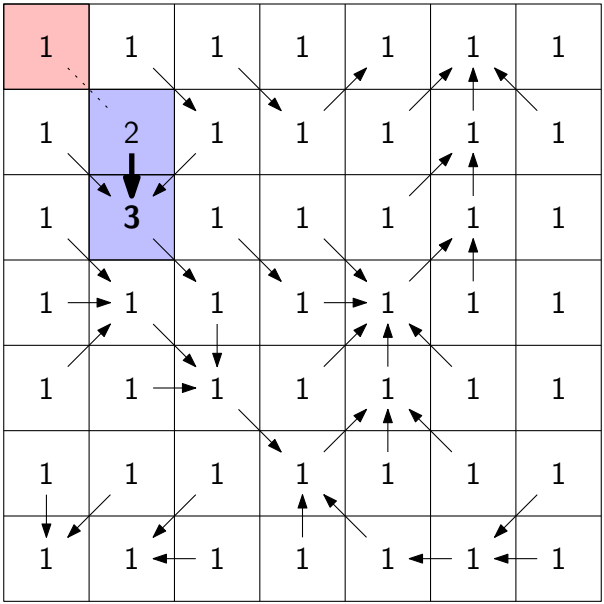
Row-by-row scan



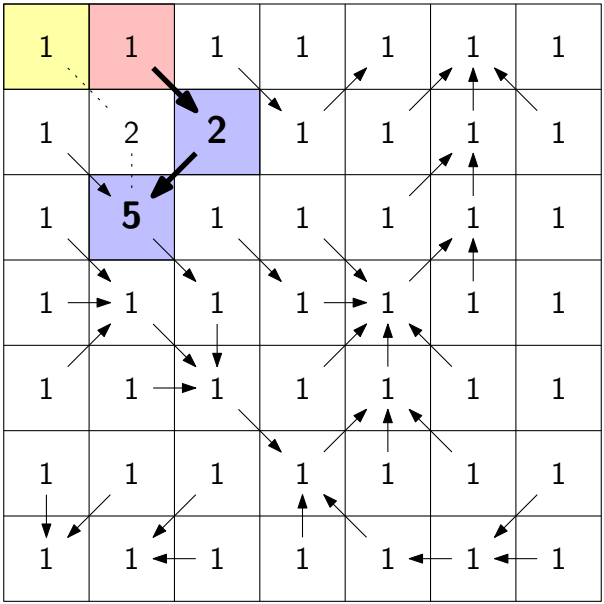
Row-by-row scan



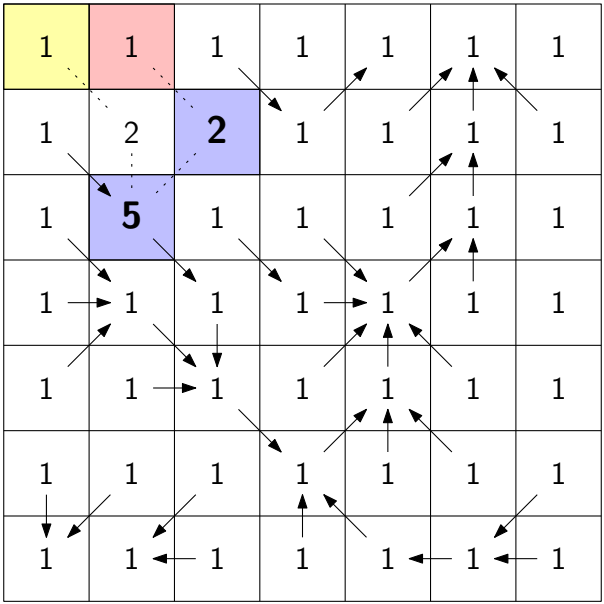
Row-by-row scan



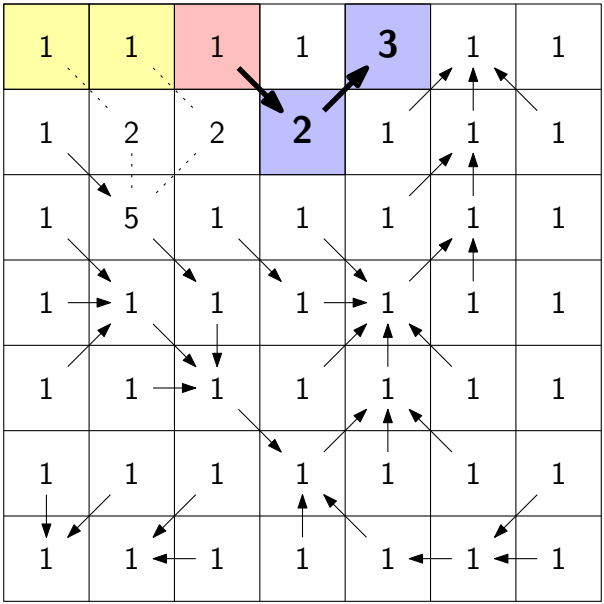
Row-by-row scan



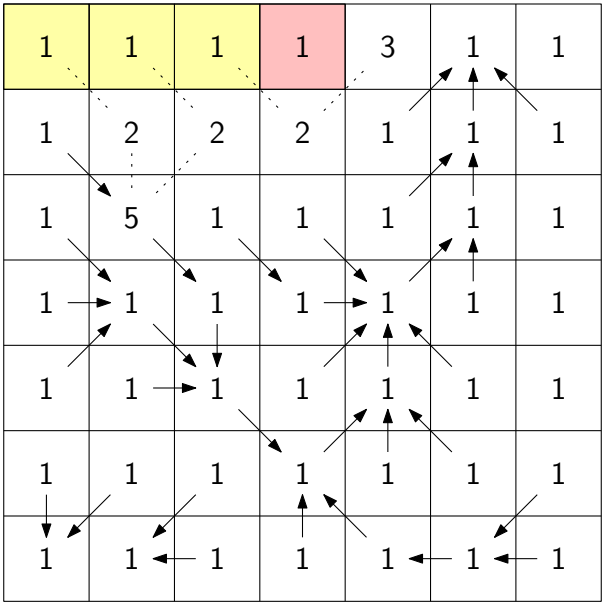
Row-by-row scan



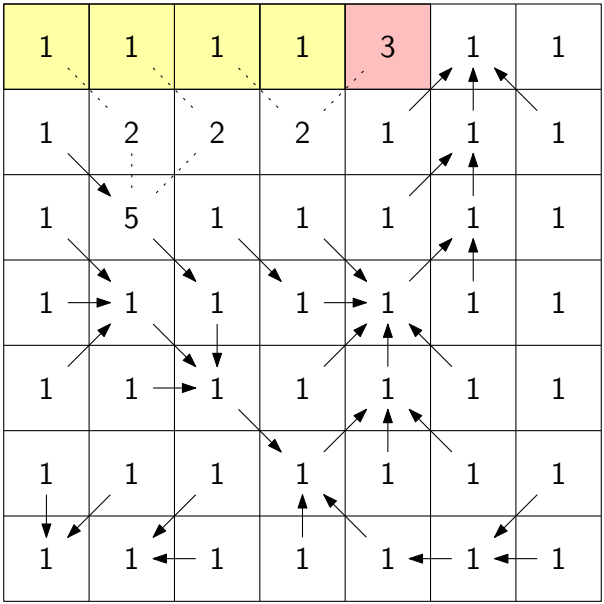
Row-by-row scan



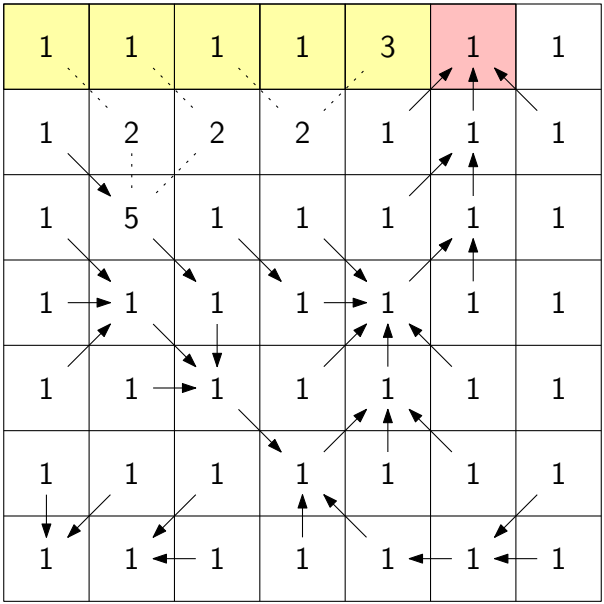
Row-by-row scan



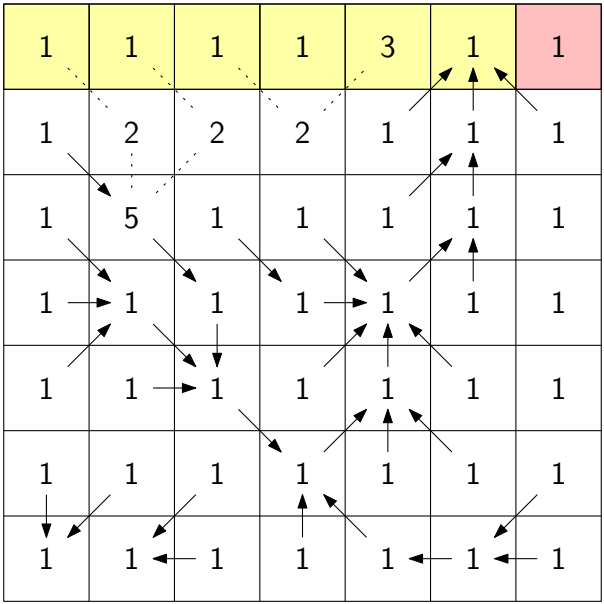
Row-by-row scan



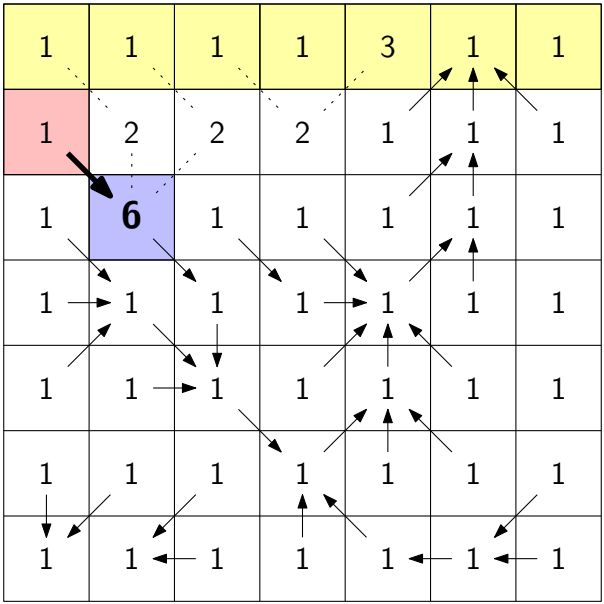
Row-by-row scan



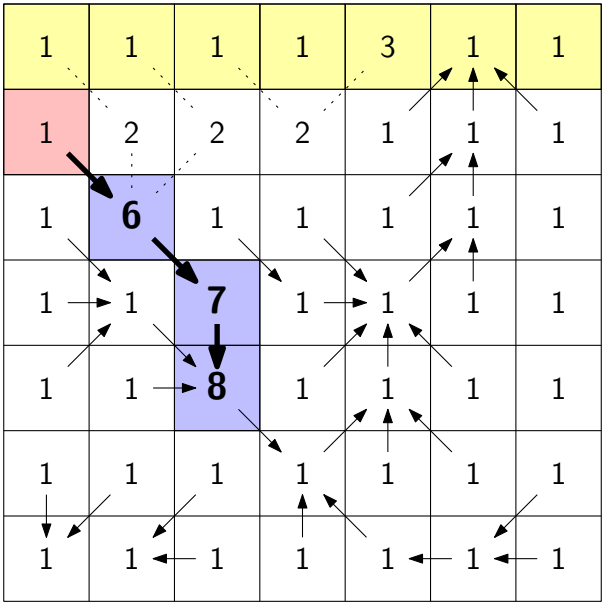
Row-by-row scan



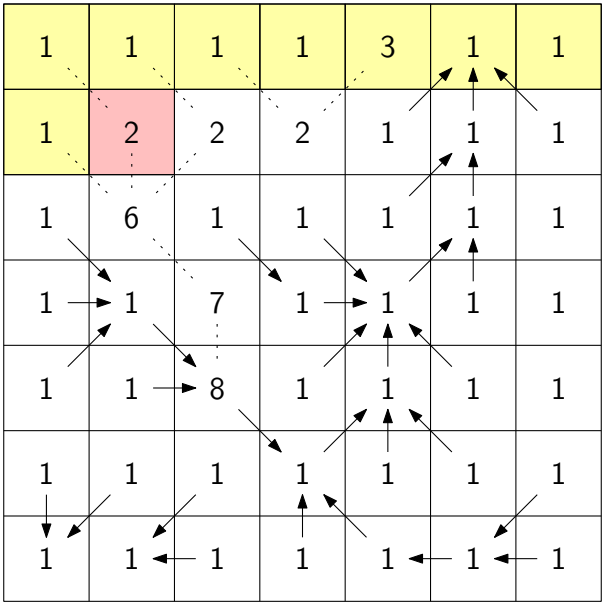
Row-by-row scan



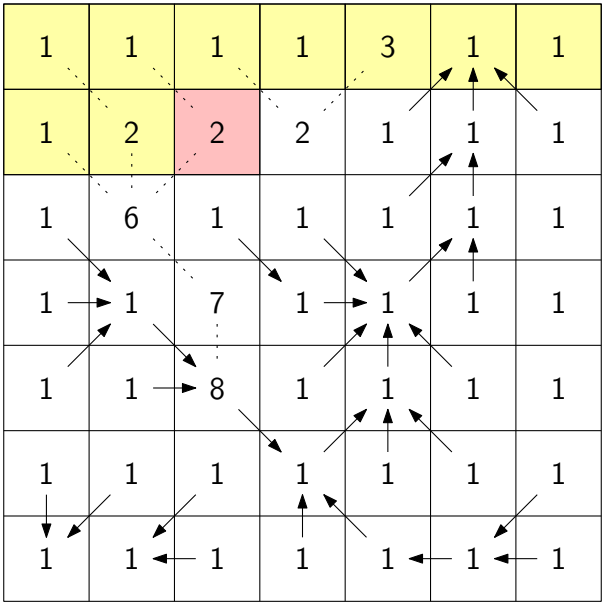
Row-by-row scan



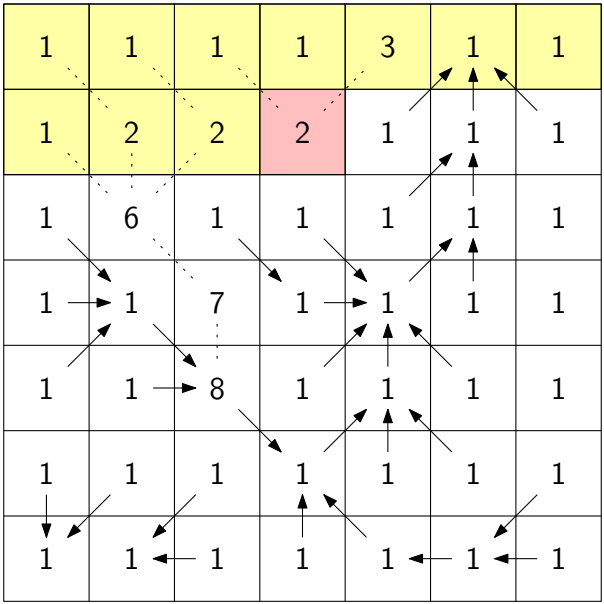
Row-by-row scan



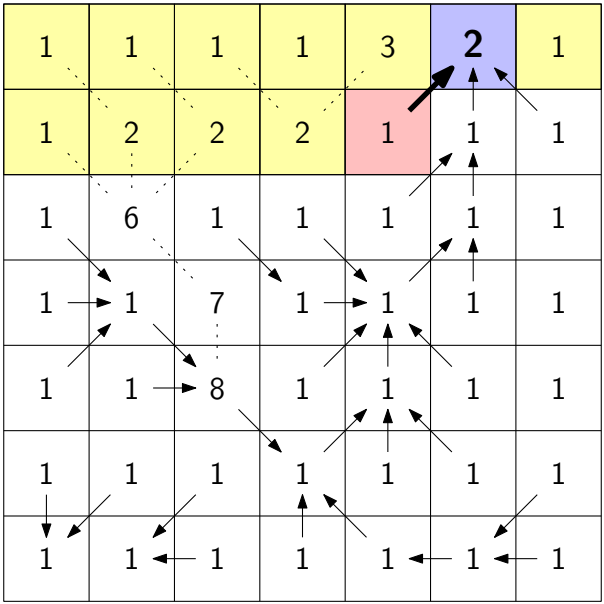
Row-by-row scan



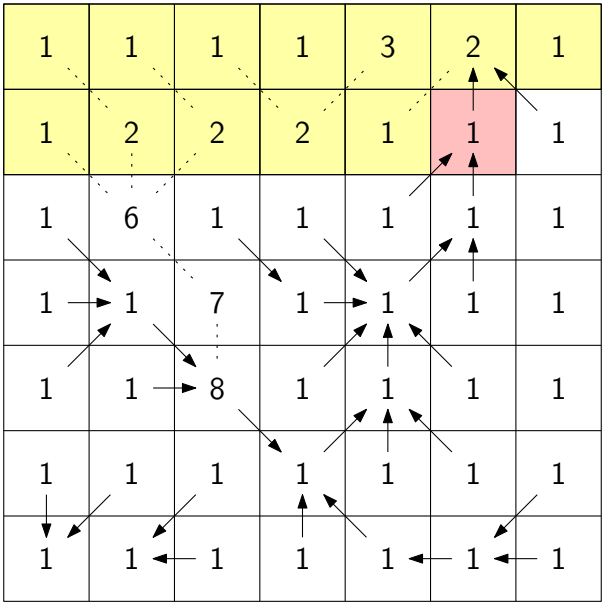
Row-by-row scan



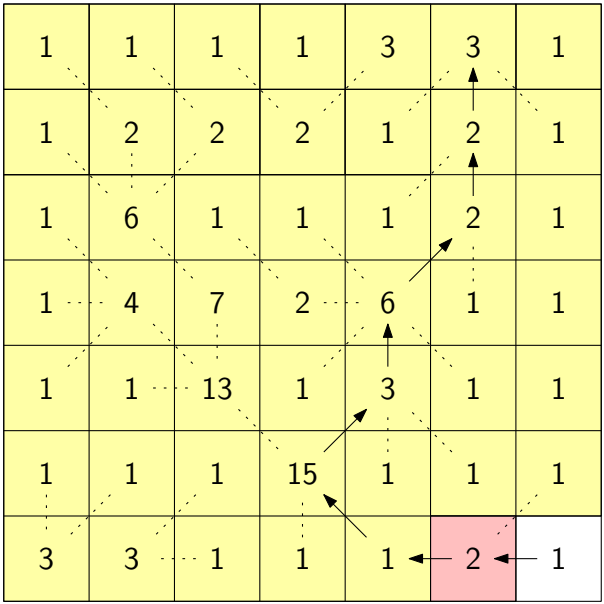
Row-by-row scan



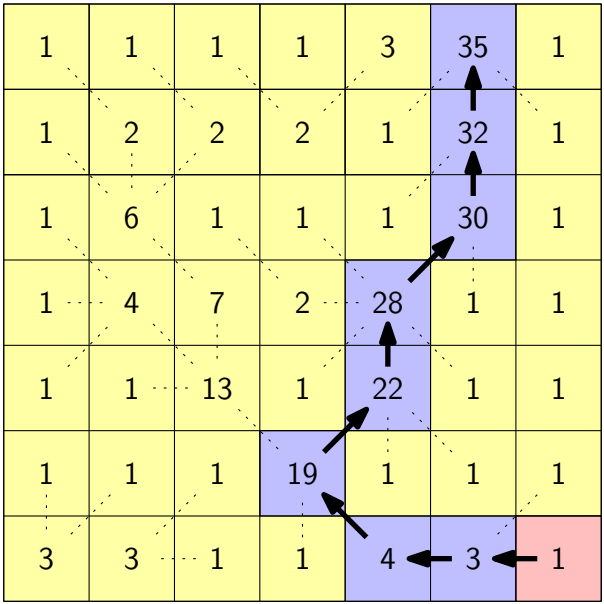
Row-by-row scan



Row-by-row scan



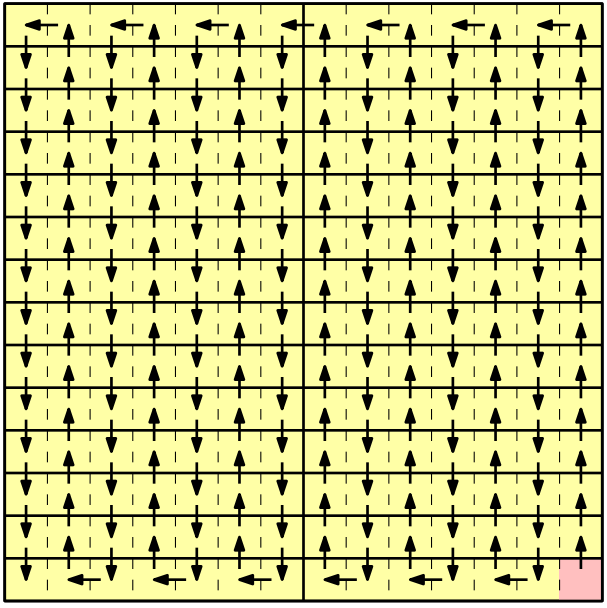
Row-by-row scan



Running time: $\Theta(n)$

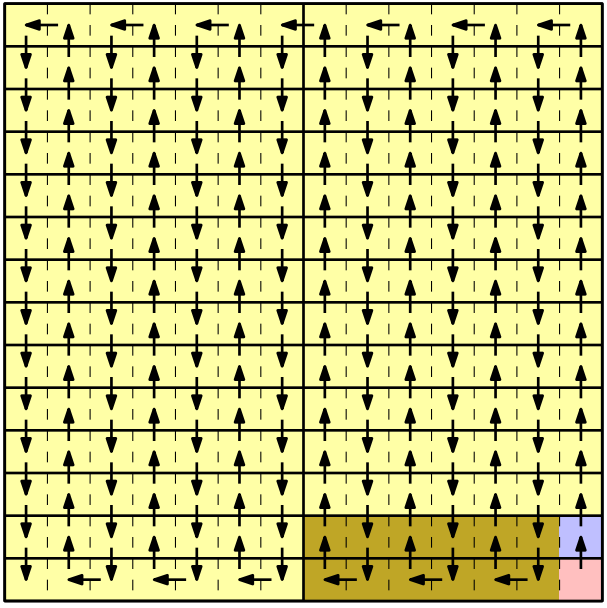
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



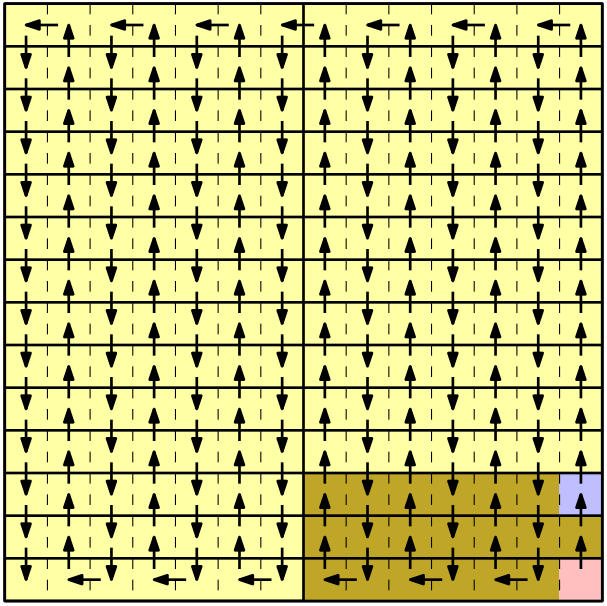
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



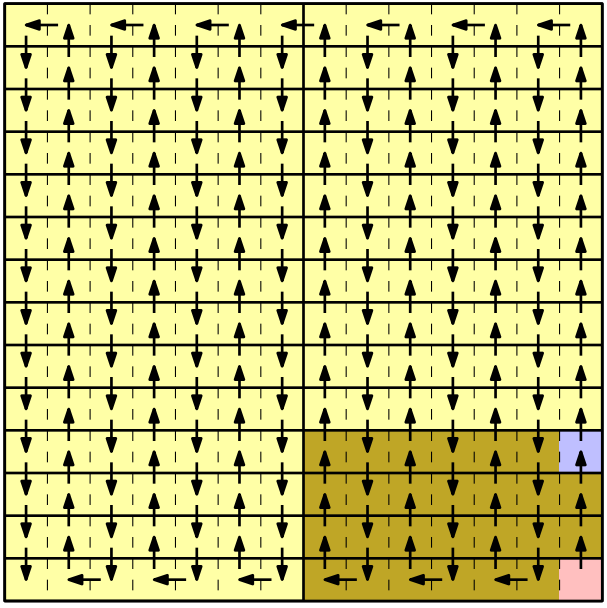
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



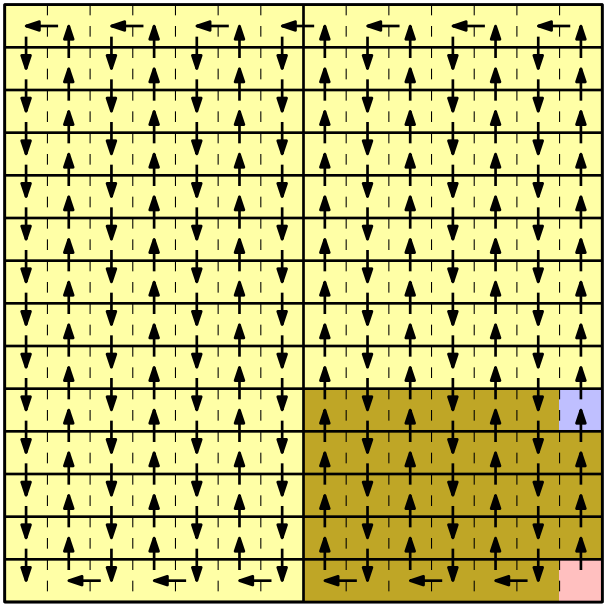
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



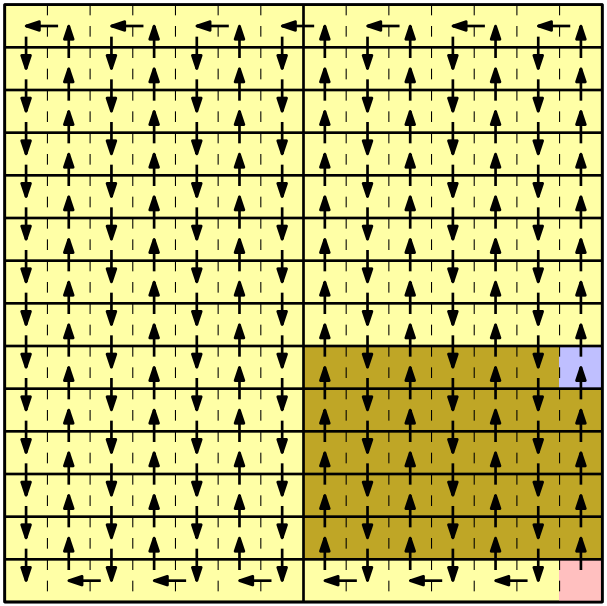
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



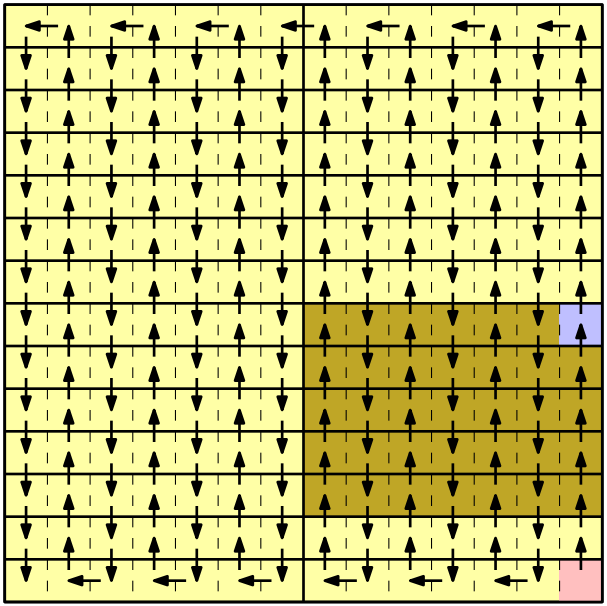
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



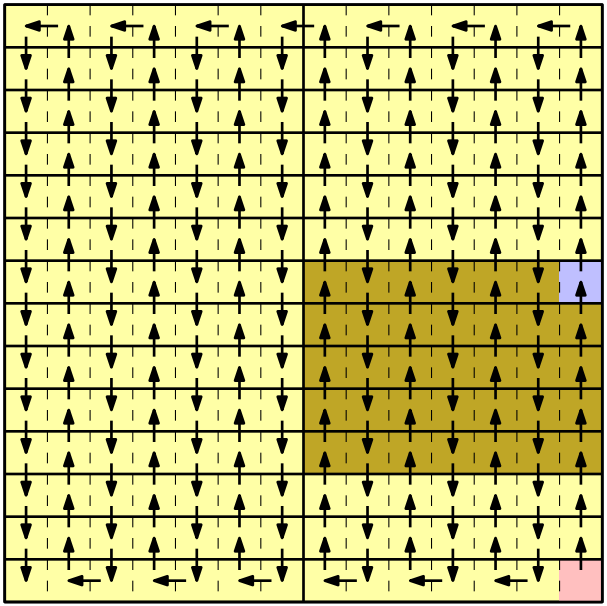
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



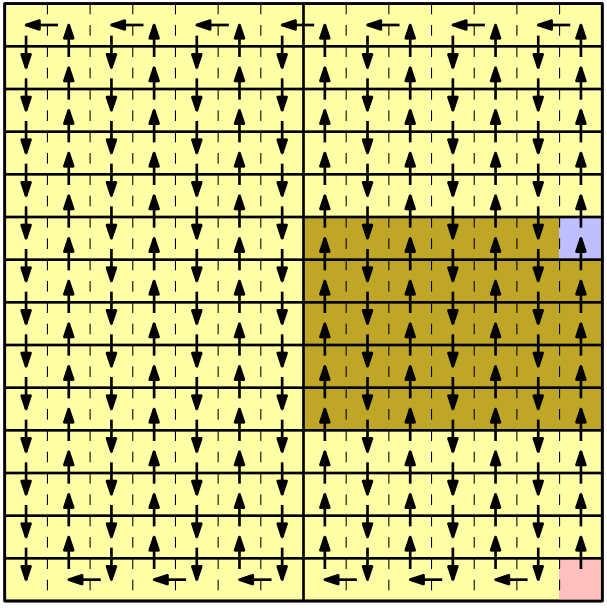
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



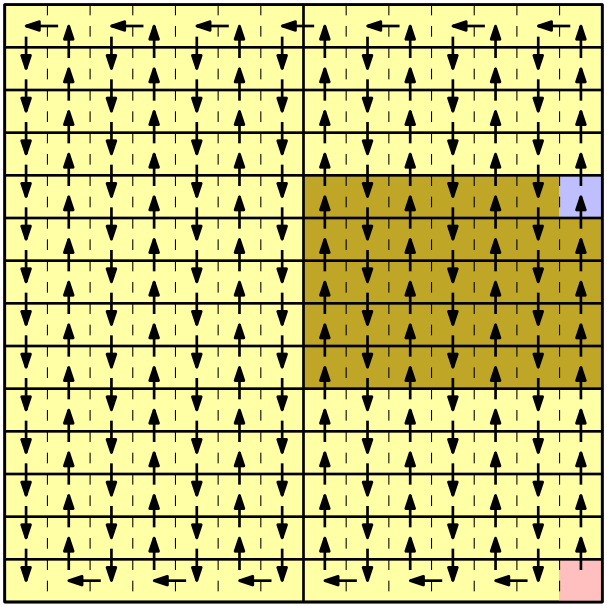
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



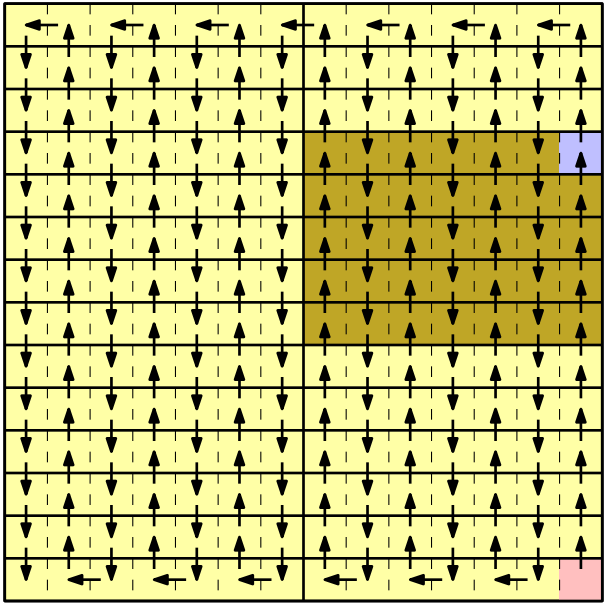
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



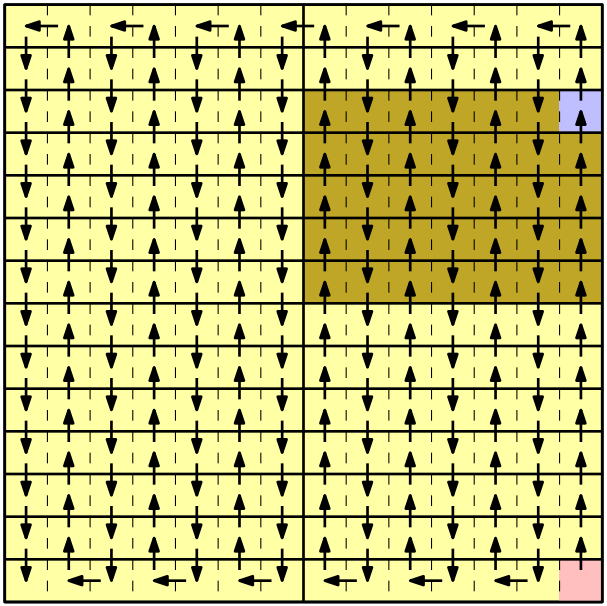
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



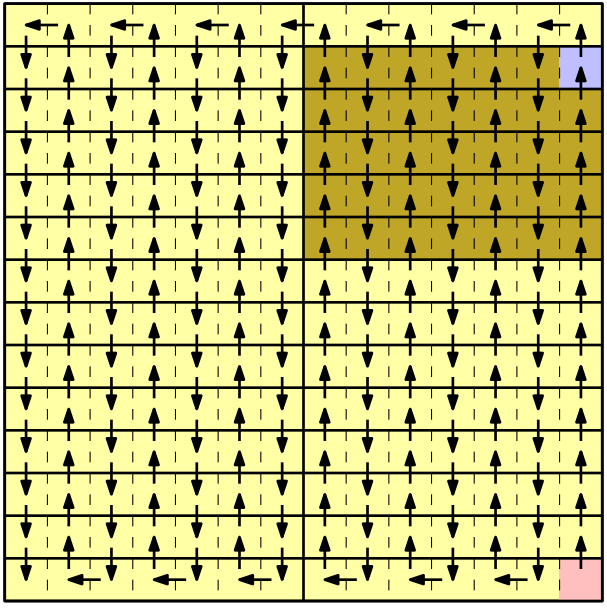
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



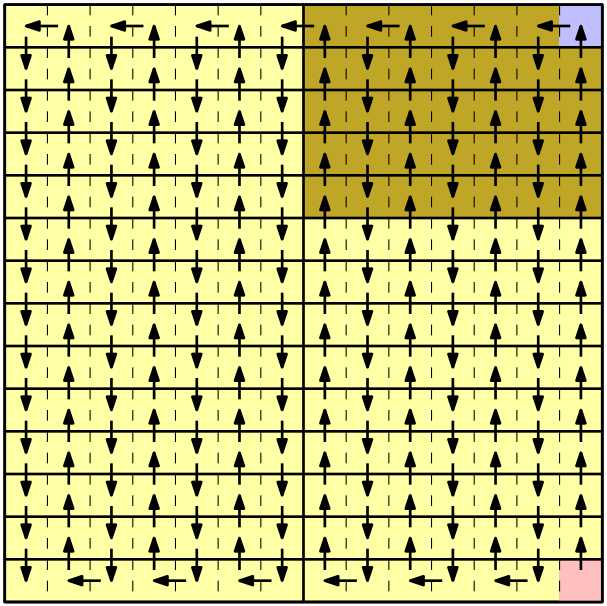
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



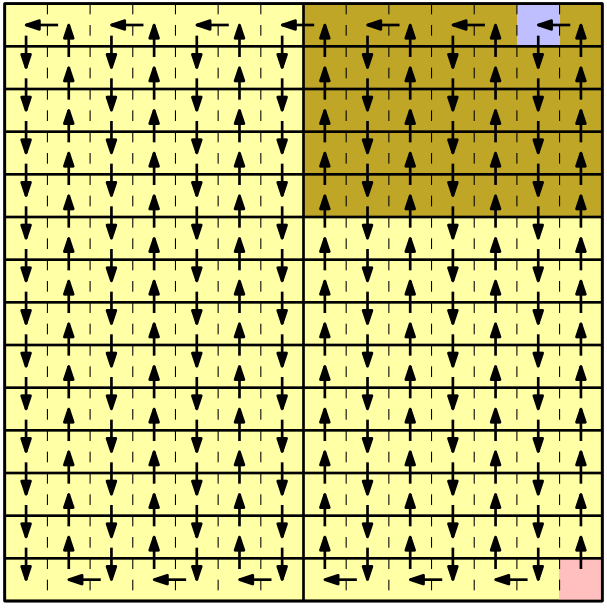
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



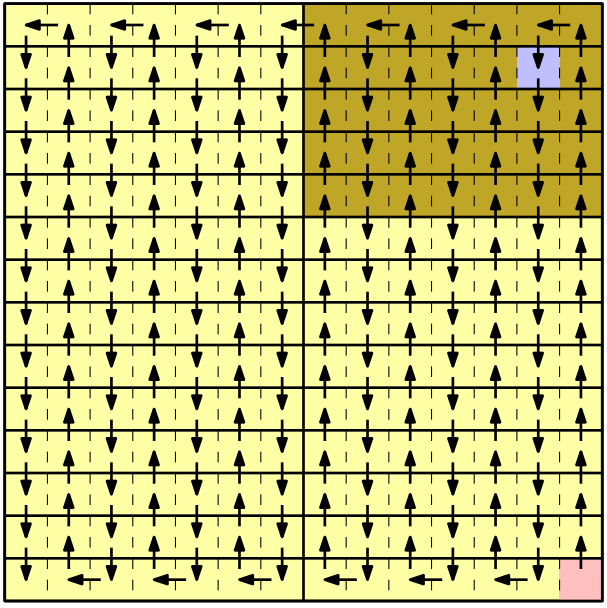
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



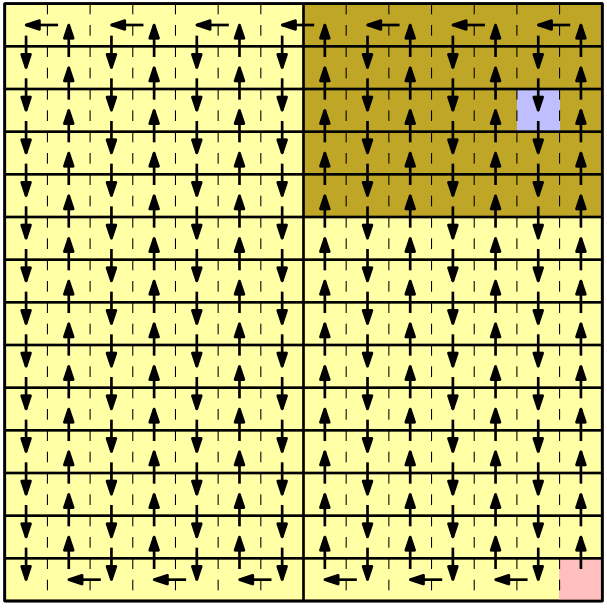
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



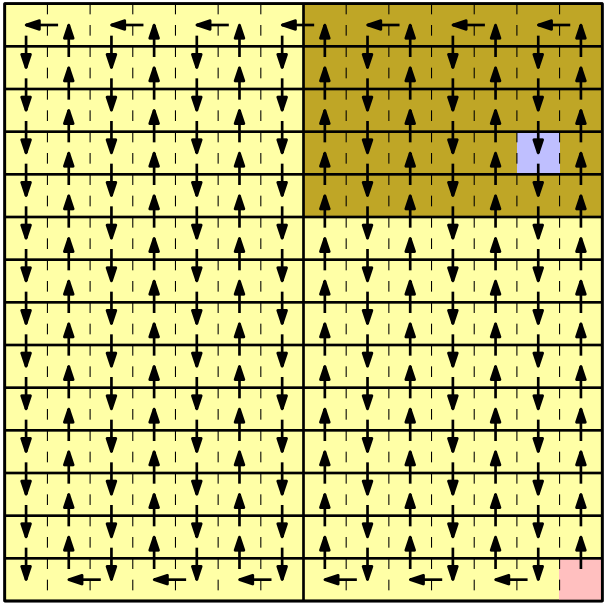
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



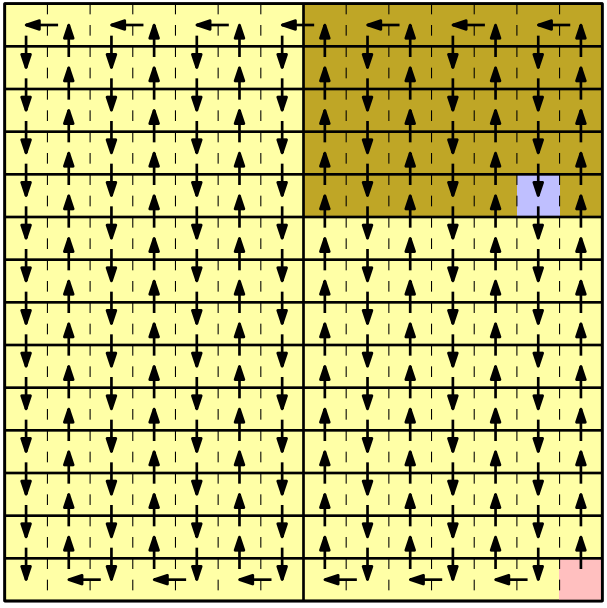
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



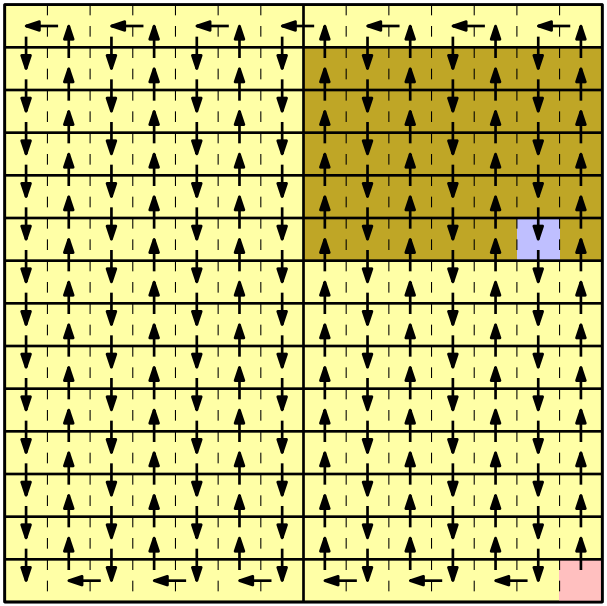
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



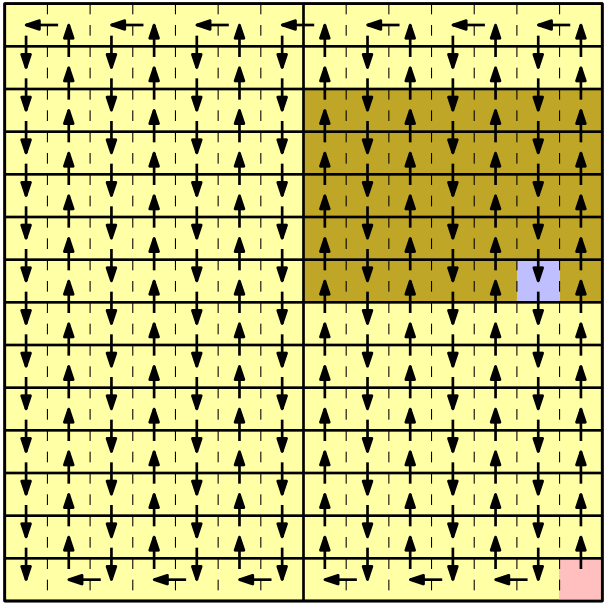
Row-by-row scan

$\Theta(N)$ I/O's in the worst case



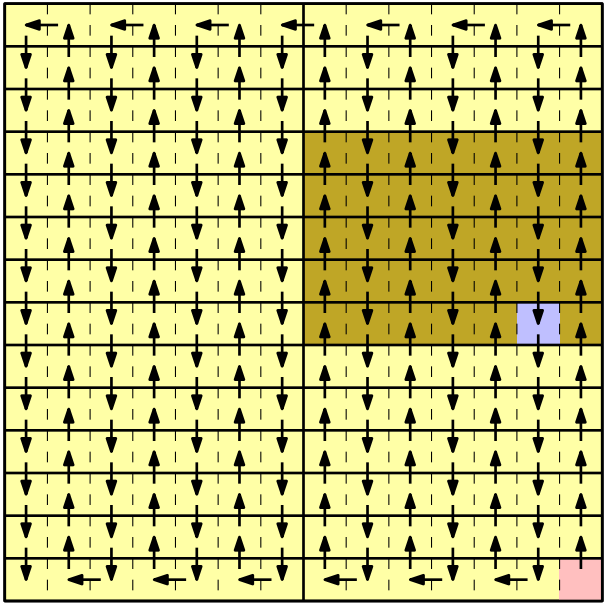
Row-by-row scan

$\Theta(N)$ I/O's in the worst case

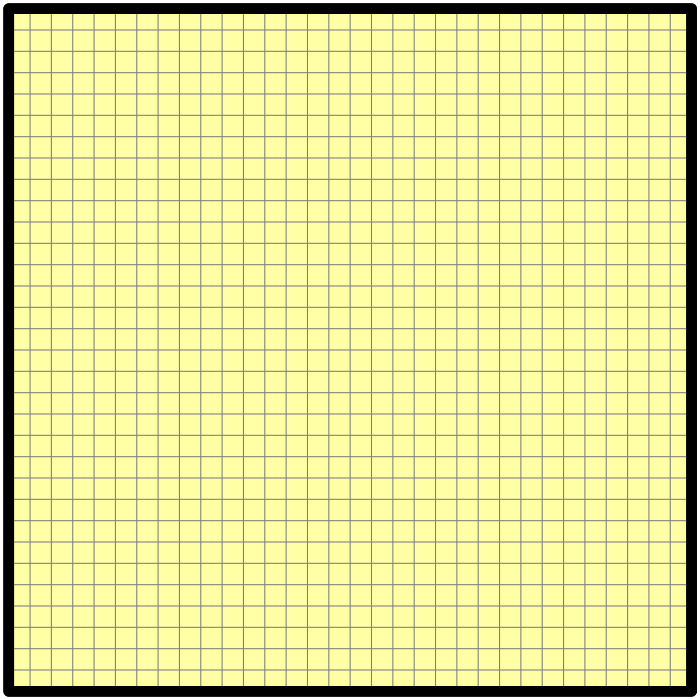


Row-by-row scan

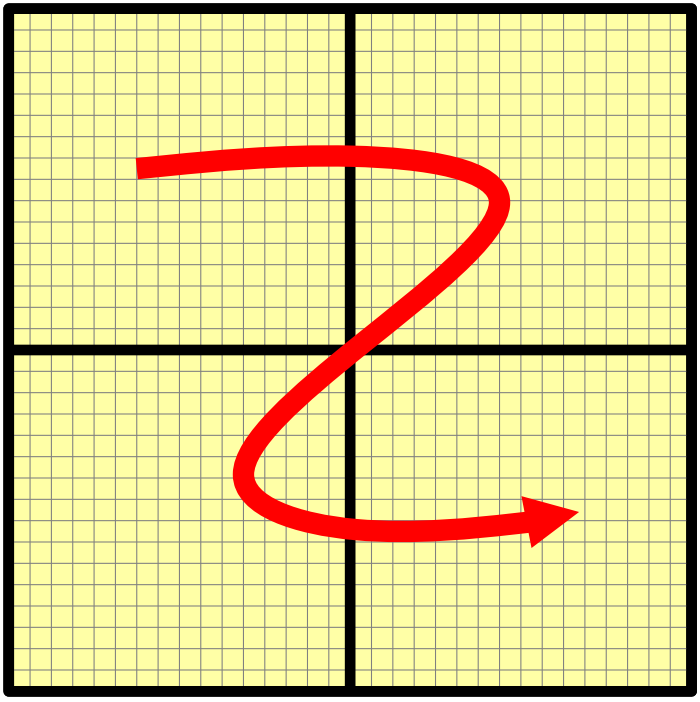
$\Theta(N)$ I/O's in the worst case



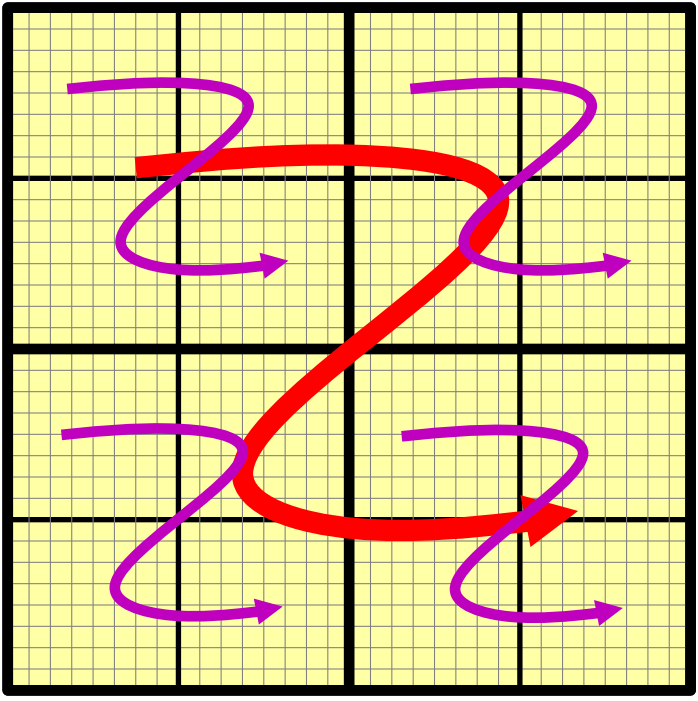
Z-order scan



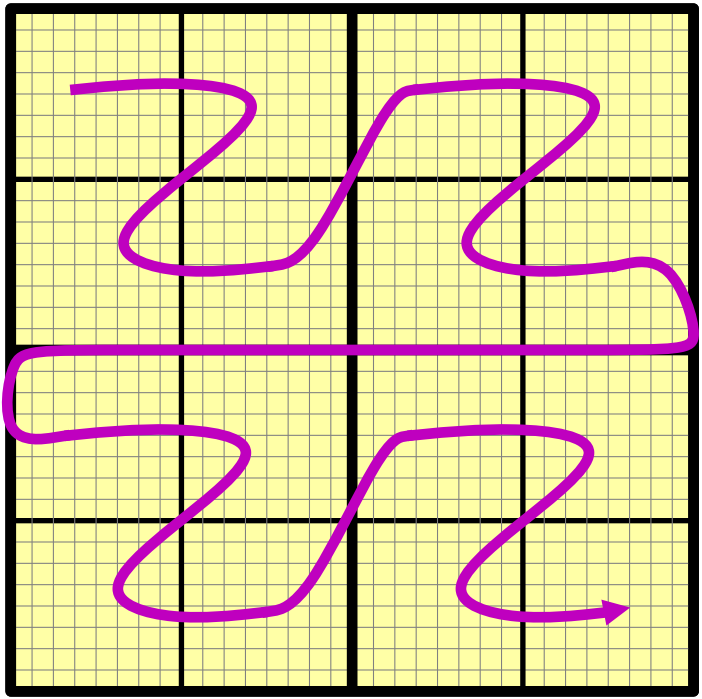
Z-order scan



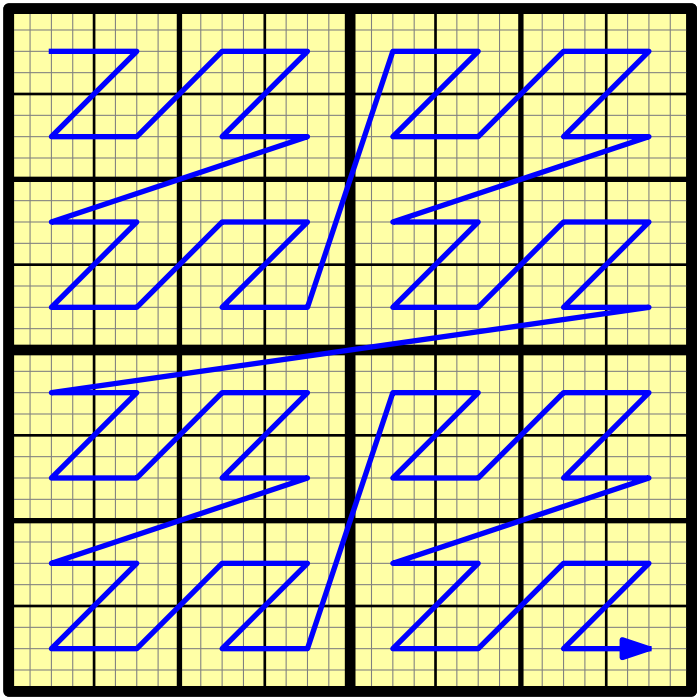
Z-order scan



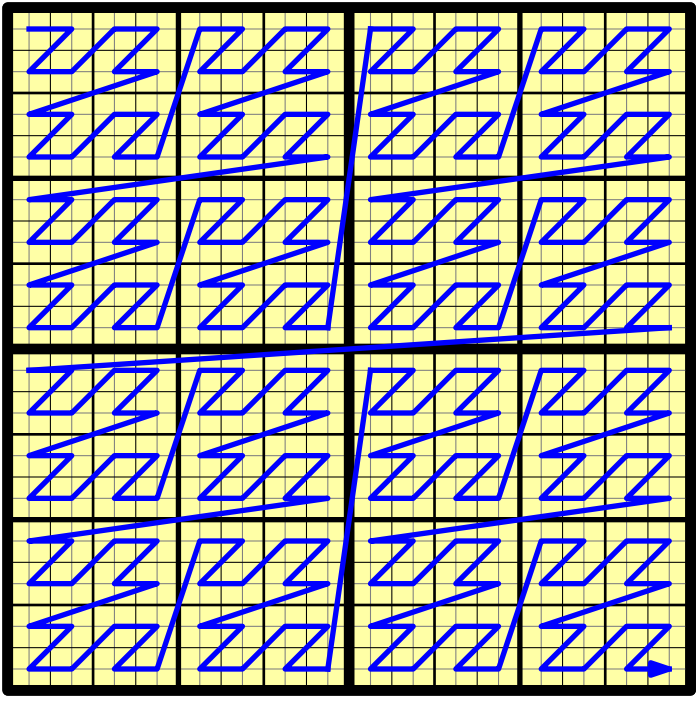
Z-order scan



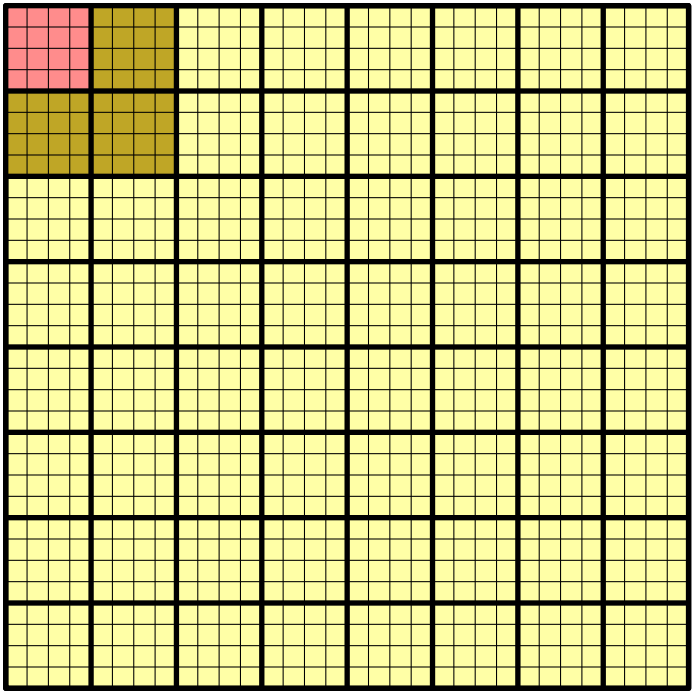
Z-order scan



Z-order scan



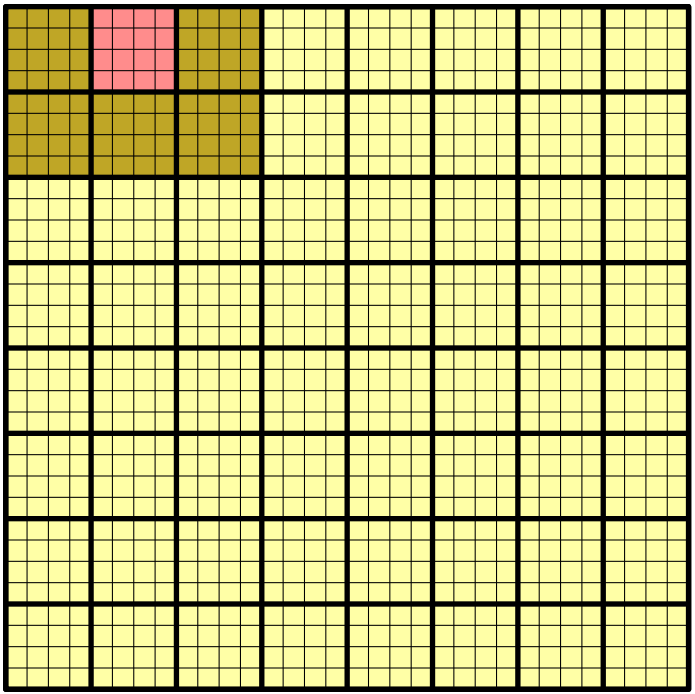
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

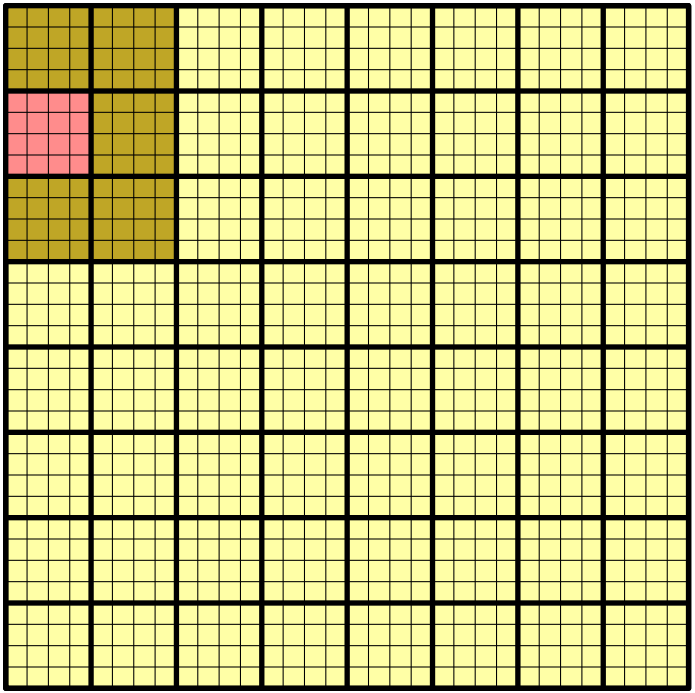
Z-order scan on Z-order file



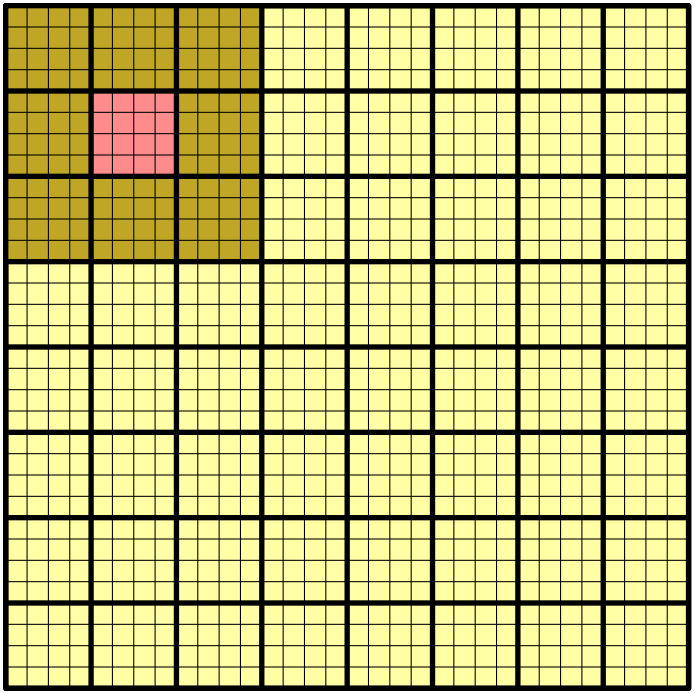
$\sqrt{B}$

While working on a block, have its neighbours in memory too

Z-order scan on Z-order file



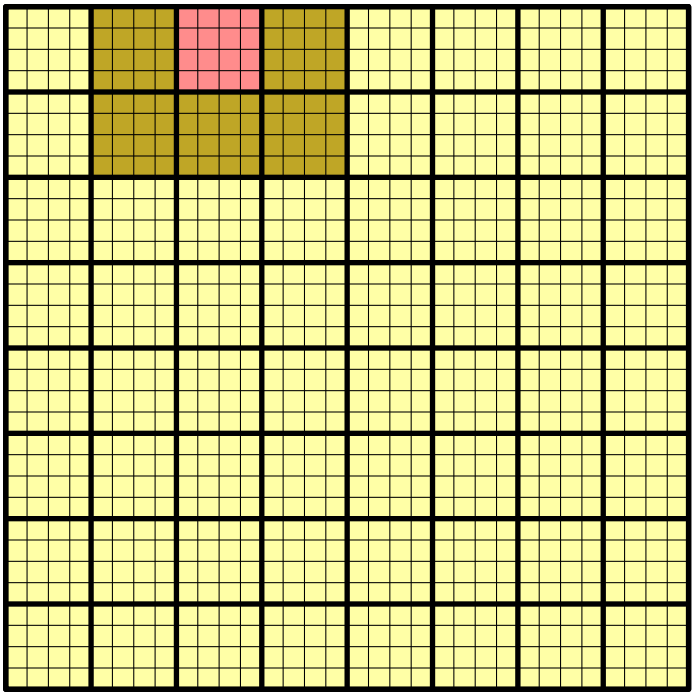
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

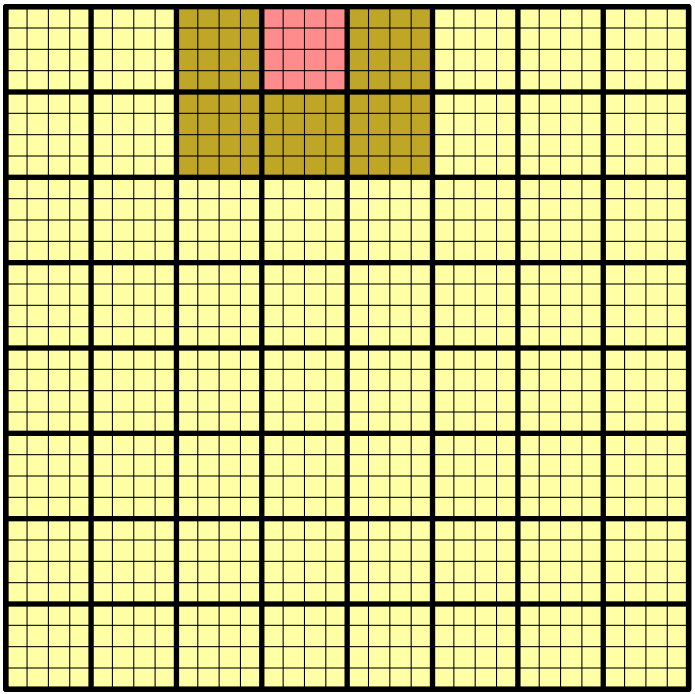
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

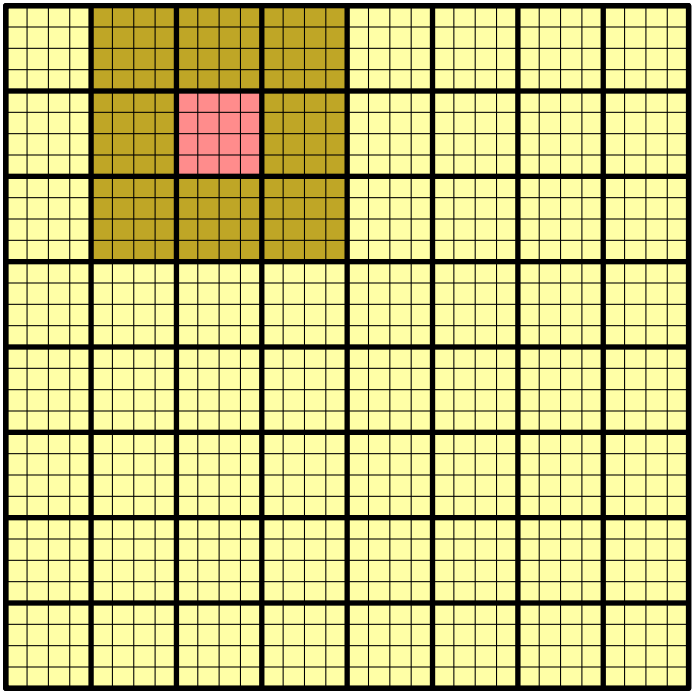
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

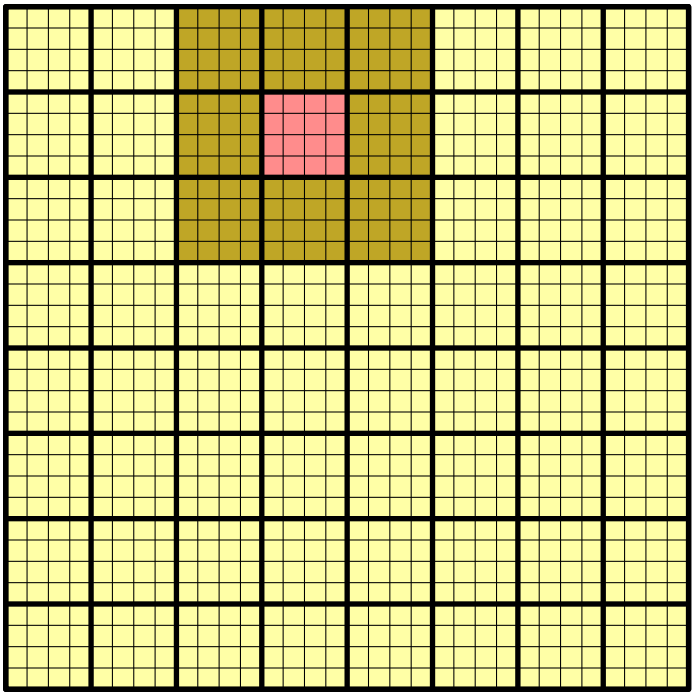
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

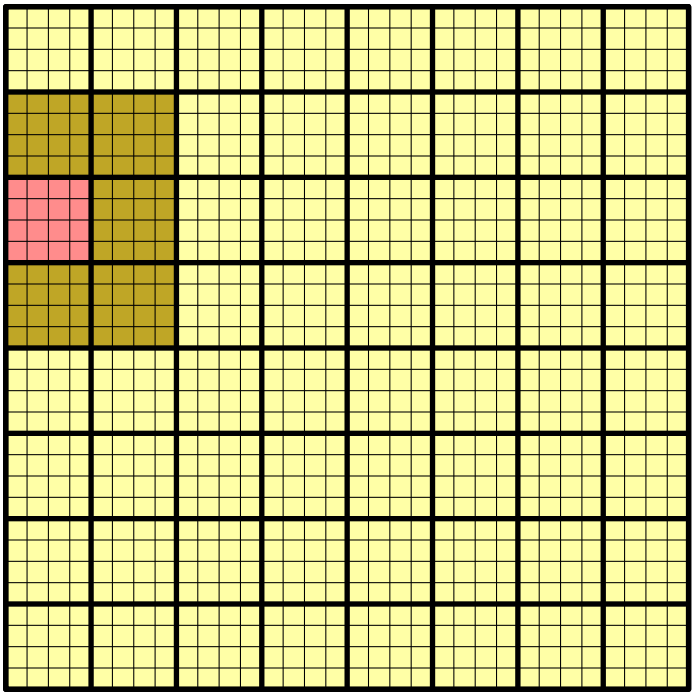
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

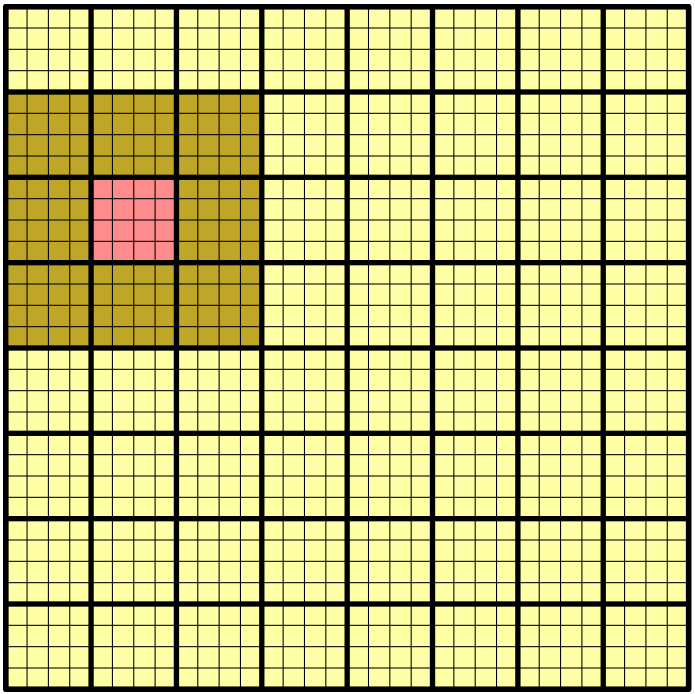
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

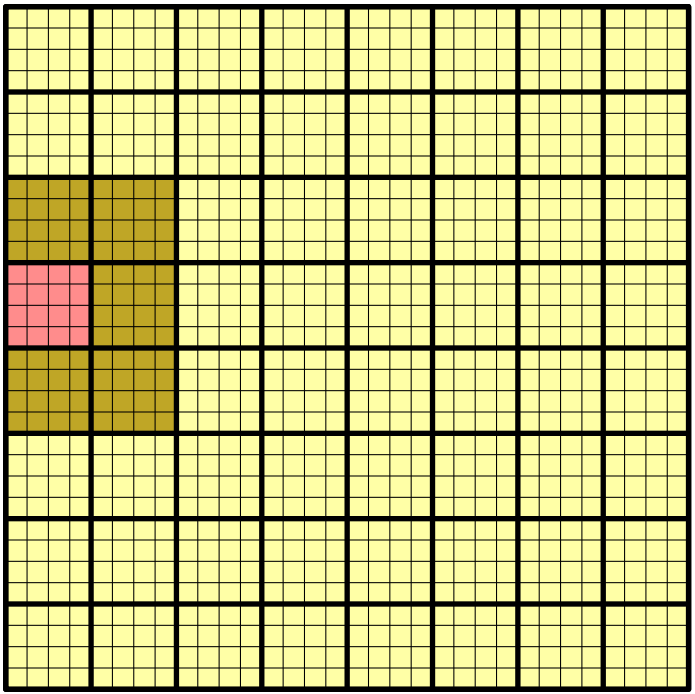
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

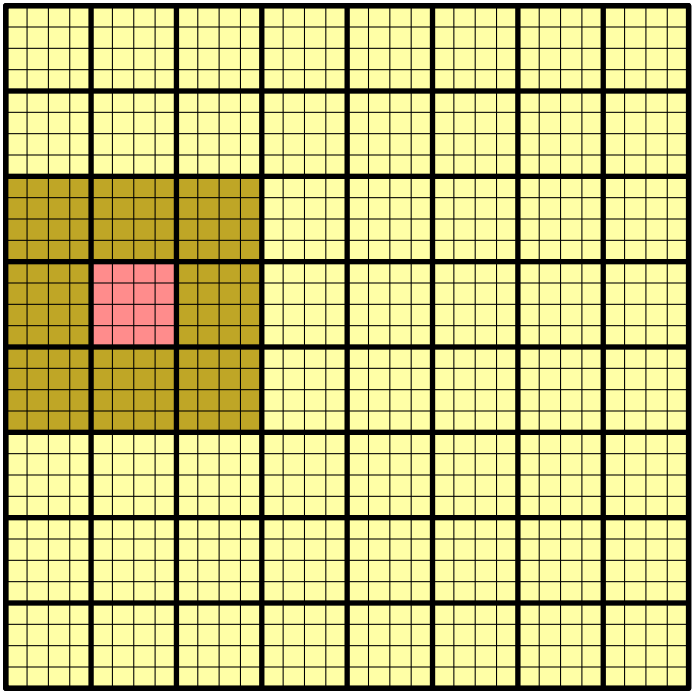
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

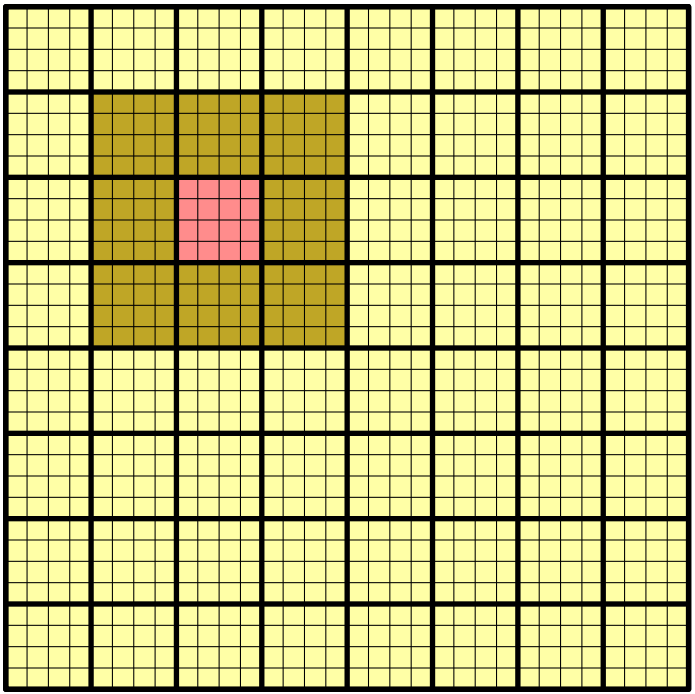
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

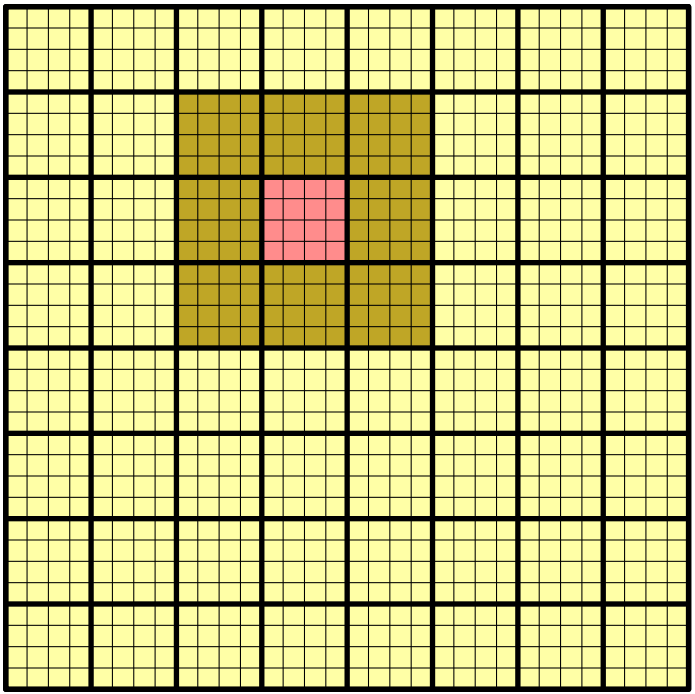
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

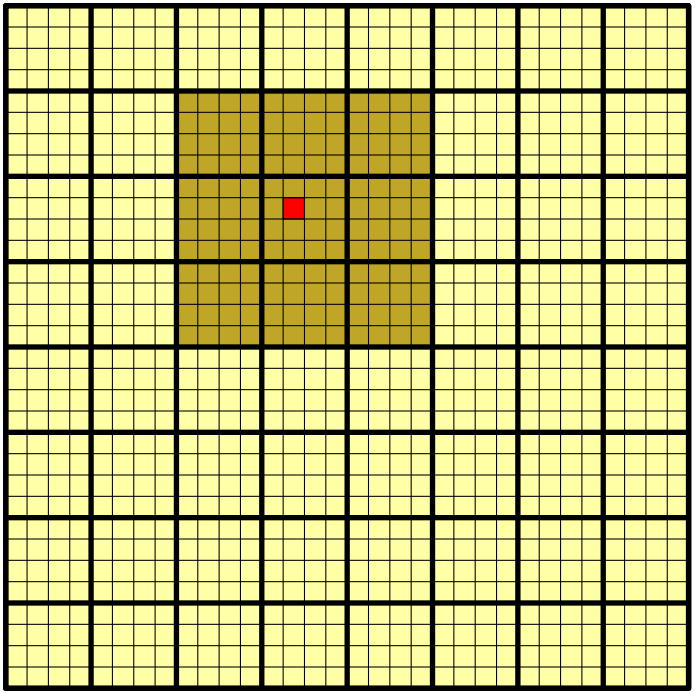
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

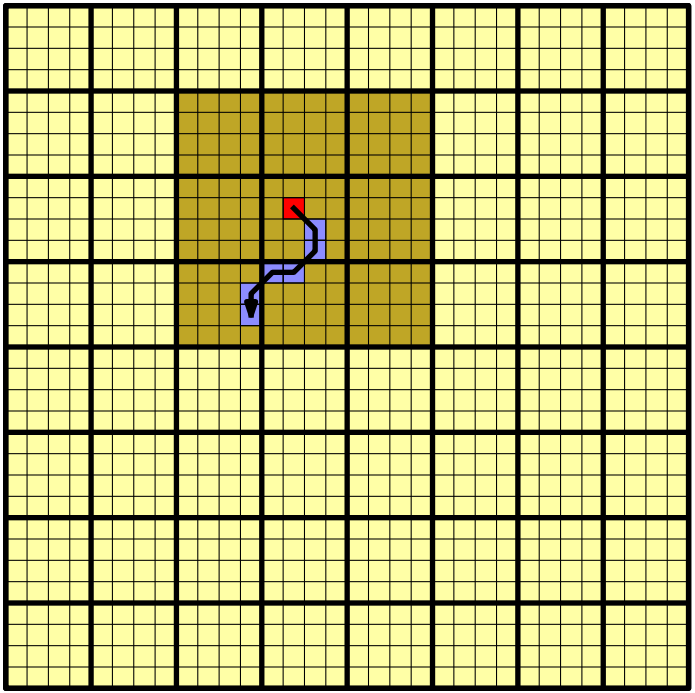
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

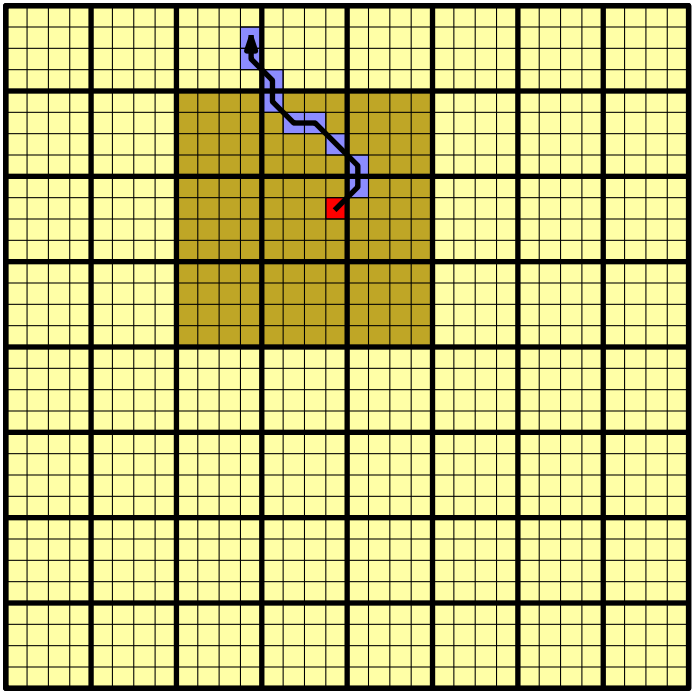
Z-order scan on Z-order file



$\sqrt{B}$

While working on a block, have its neighbours in memory too

Z-order scan on Z-order file



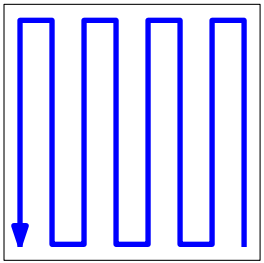
$\sqrt{B}$

While working on a block, have its neighbours in memory too

Only long paths require additional swapping

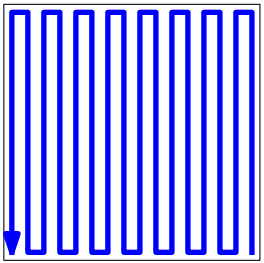
Worst-case terrains vs. real terrains

Worst-case, size $n = 8^2$

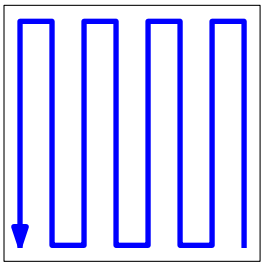


$\Omega(\sqrt{N})$ big turns

Worst-case, size $4n$

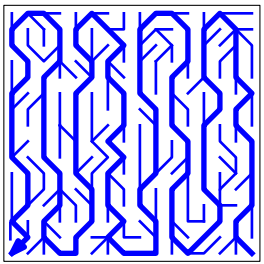


Realistic, size $n = 8^2$



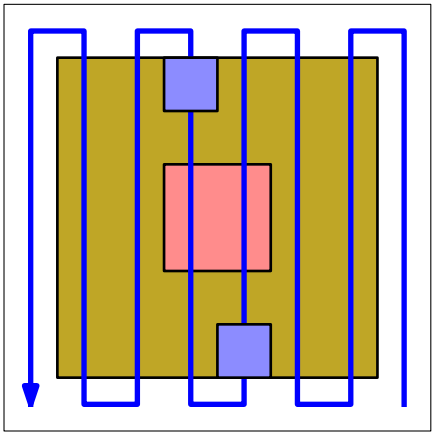
$\Theta(1)$ big turns

Realistic, size $4n$

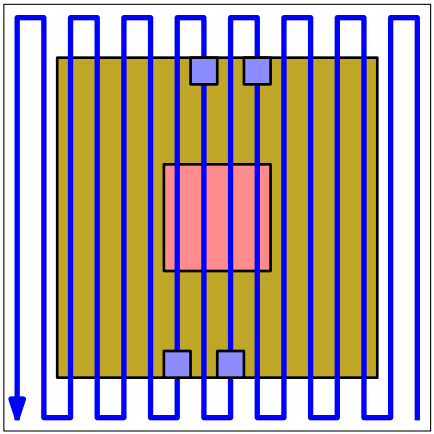


Worst-case terrains vs. real terrains

Worst-case, size $n = 8^2$



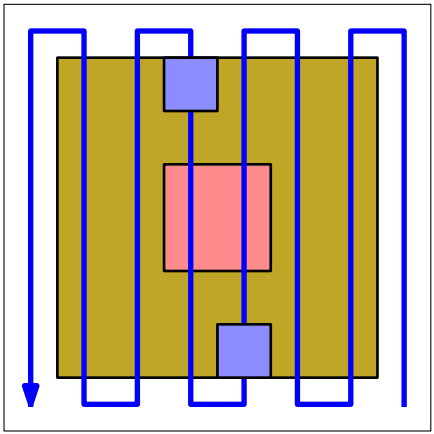
Worst-case, size $4n$



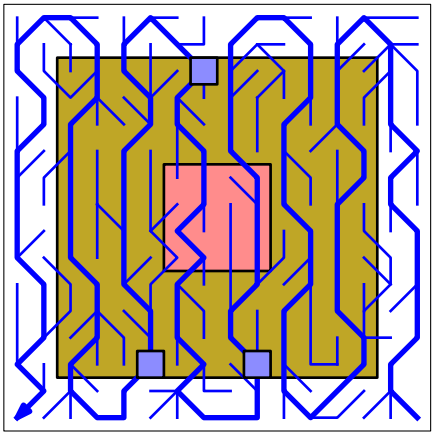
$Q' = Q$ scaled by factor 3.
Far cells of Q: cells on boundary of Q' where water from Q collects.
In the worst case, maximum number of far cells grows with resolution.

Worst-case terrains vs. real terrains

Realistic, size $n = 8^2$

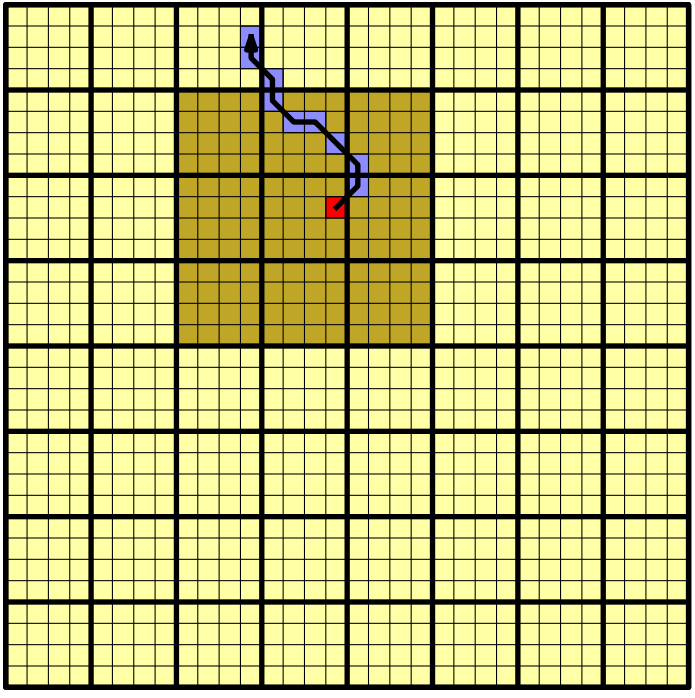


Realistic, size $4n$



$Q' = Q$ scaled by factor 3.
Far cells of Q: cells on boundary of Q' where water from Q collects.
Confluence assumption: number of far cells for any square $Q \leq$ constant γ

Z-order scan on Z-order file



$\sqrt{B}$

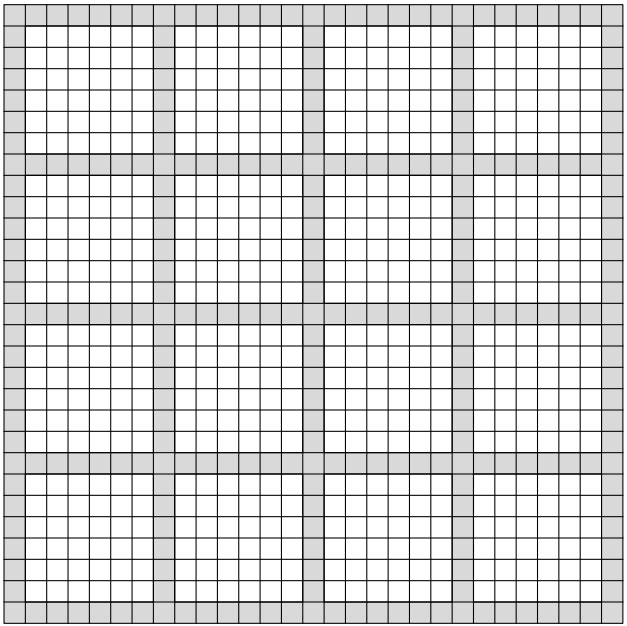
While working on a block, have its neighbours in memory too

$\Theta(\mathrm{scan}(N))$I/Os

Only long paths require additional swapping

N/B blocks $\times$ γ swaps $\times$ 9-block window $=$ $\Theta(\mathrm{scan}(N))$I/Os

Cache-aware separator-based algorithm



Separator cells: each $\Theta(\sqrt{M})$-th row/column

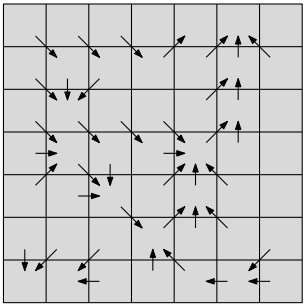
$\rightarrow$

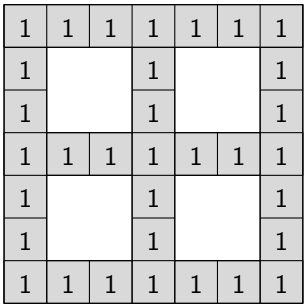
divide grid into $\Theta(N/M)$ subgrids of size $\Theta(M)$

Cache-aware separator-based algorithm

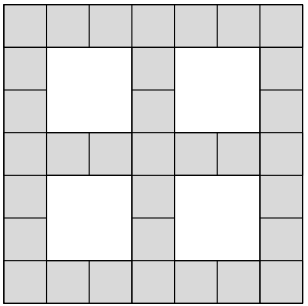
input

1. move flow from interior of subgrids to separators





separator flow accumul.

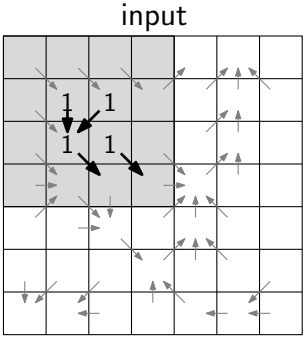


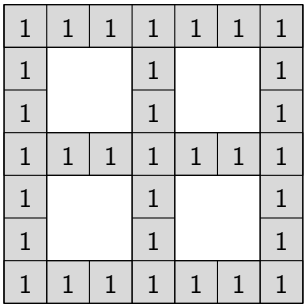
$\Theta(\sqrt{M})$

separator flow directions

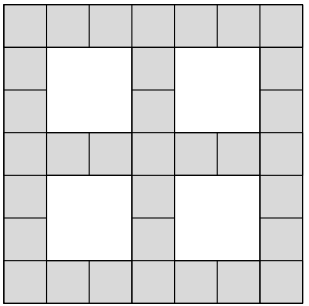
Cache-aware separator-based algorithm

1. move flow from interior of subgrids to separators





separator flow accumul.

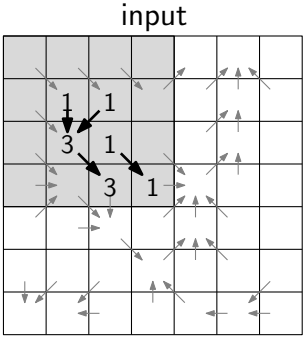


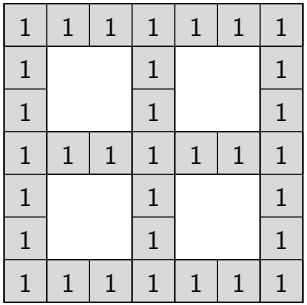
$\Theta(\sqrt{M})$

separator flow directions

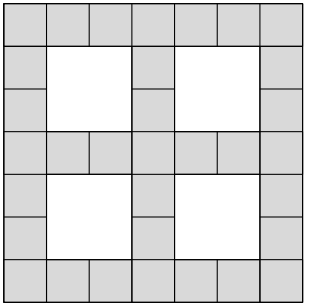
Cache-aware separator-based algorithm

1. move flow from interior of subgrids to separators





separator flow accumul.



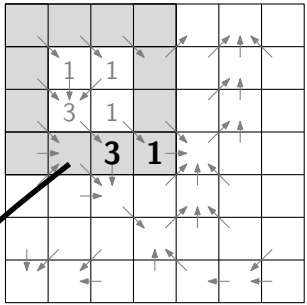
$\Theta(\sqrt{M})$

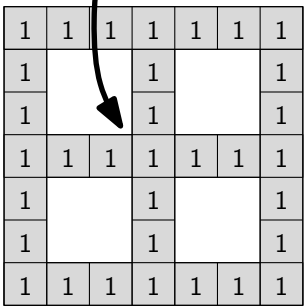
separator flow directions

Cache-aware separator-based algorithm

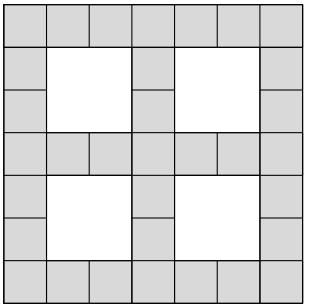
input

1. move flow from interior of subgrids to separators





separator flow accumul.



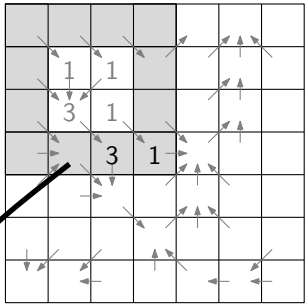
$\Theta(\sqrt{M})$

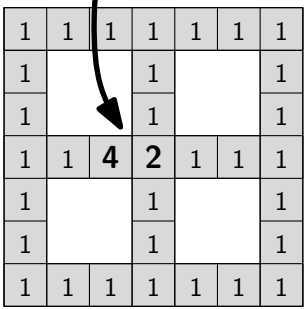
separator flow directions

Cache-aware separator-based algorithm

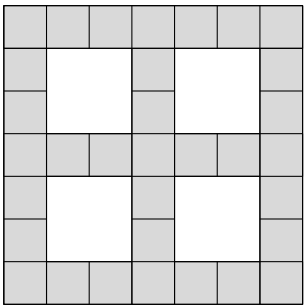
input

1. move flow from interior of subgrids to separators





separator flow accumul.



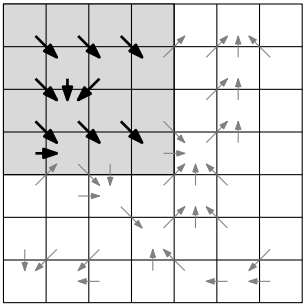
$\Theta(\sqrt{M})$

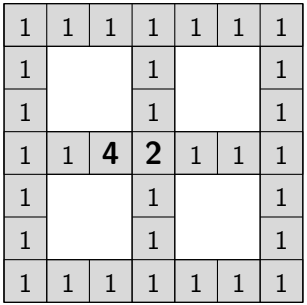
separator flow directions

Cache-aware separator-based algorithm

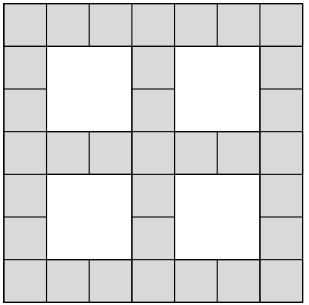
input

1. move flow from interior of subgrids to separators and compute flow connections between separators





separator flow accumul.

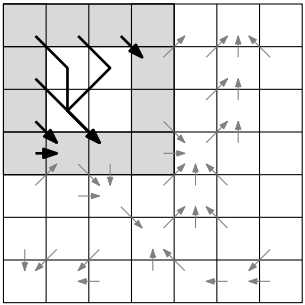


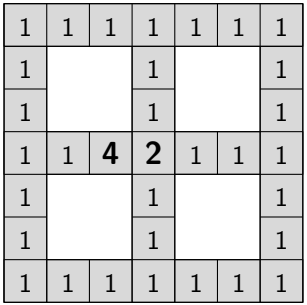
separator flow directions

Cache-aware separator-based algorithm

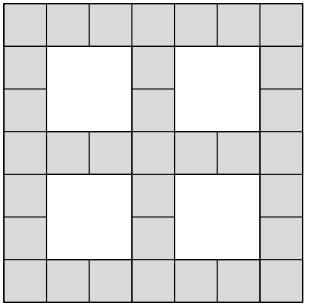
input

1. move flow from interior of subgrids to separators and compute flow connections between separators





separator flow accumul.



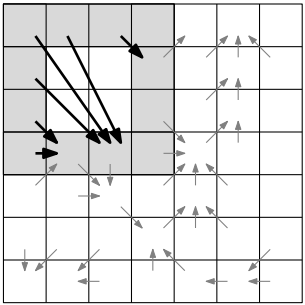
$\Theta(\sqrt{M})$

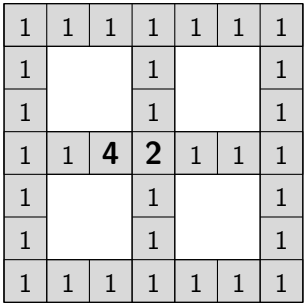
separator flow directions

Cache-aware separator-based algorithm

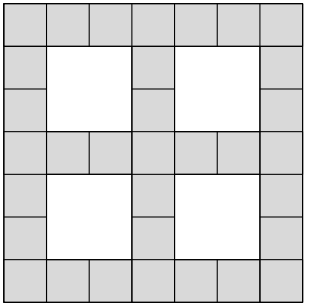
input

1. move flow from interior of subgrids to separators and compute flow connections between separators





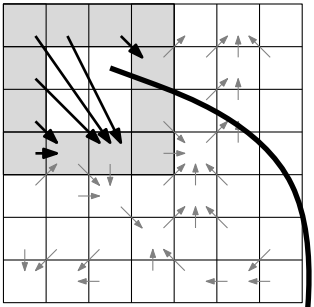
separator flow accumul.

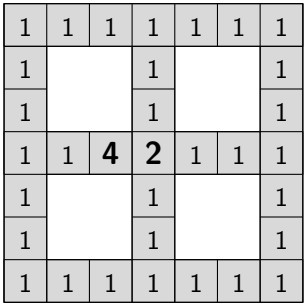


separator flow directions

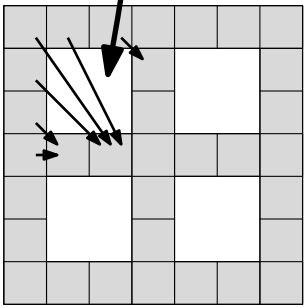
Cache-aware separator-based algorithm

1. move flow from interior of subgrids to separators and compute flow connections between separators





separator flow accumul.

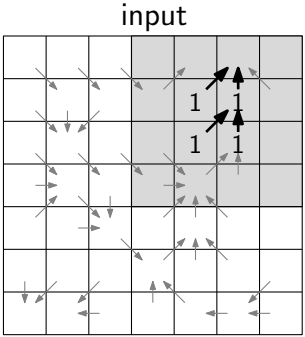


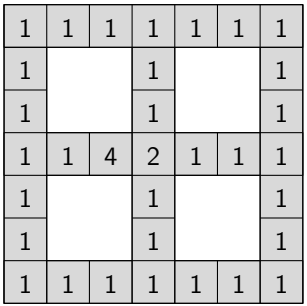
$\Theta(\sqrt{M})$

separator flow directions

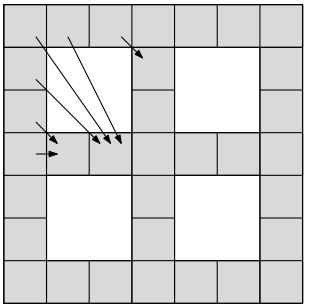
Cache-aware separator-based algorithm

1. move flow from interior of subgrids to separators and compute flow connections between separators





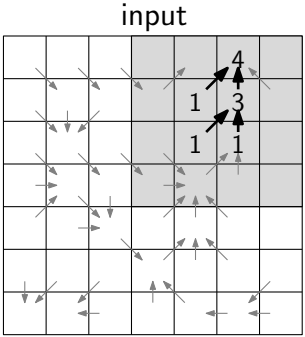
separator flow accumul.

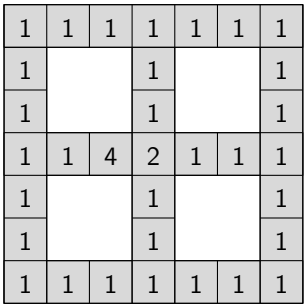


separator flow directions

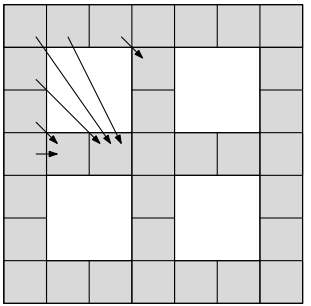
Cache-aware separator-based algorithm

1. move flow from interior of subgrids to separators and compute flow connections between separators





separator flow accumul.

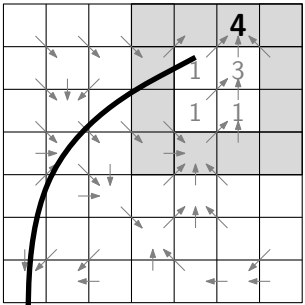


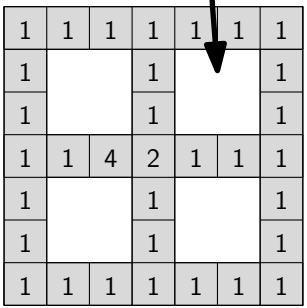
separator flow directions

Cache-aware separator-based algorithm

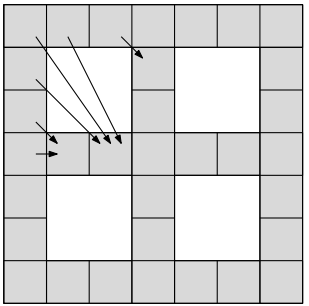
input

1. move flow from interior of subgrids to separators and compute flow connections between separators





separator flow accumul.



$\Theta(\sqrt{M})$

separator flow directions

Cache-aware separator-based algorithm

input

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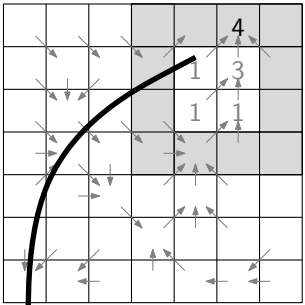
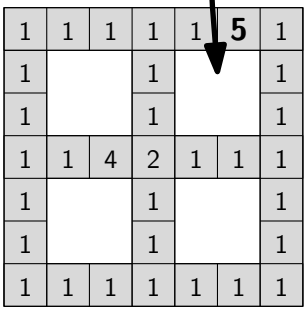
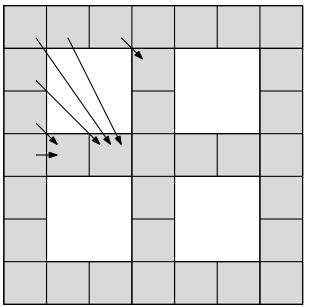




separator flow accumul.



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separator flow directions

Cache-aware separator-based algorithm

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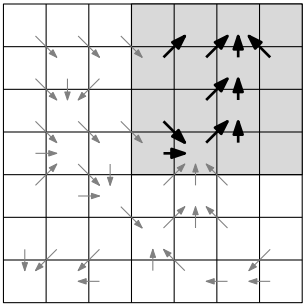

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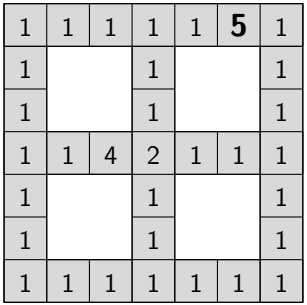
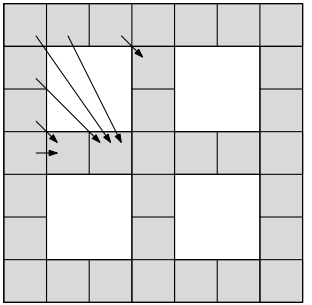


separator flow accumul.



separator flow directions

Cache-aware separator-based algorithm

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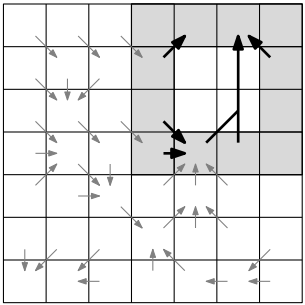

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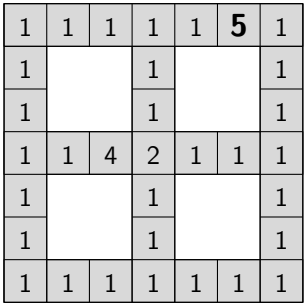

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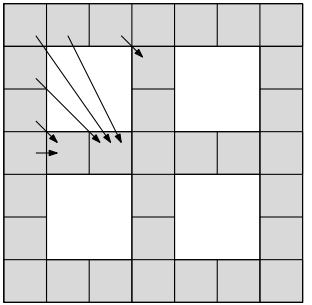

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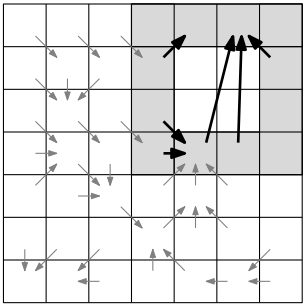

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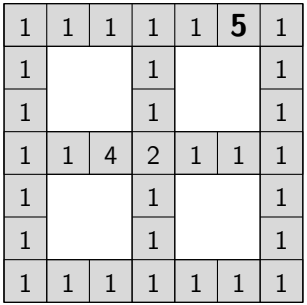
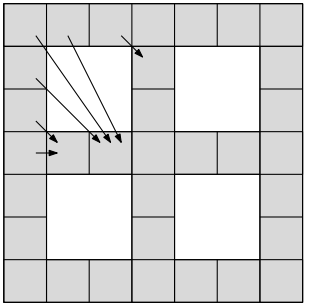


separator flow accumul.



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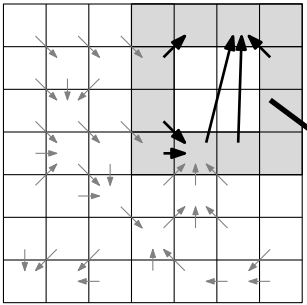
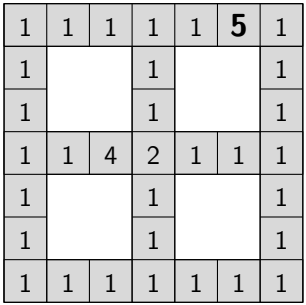
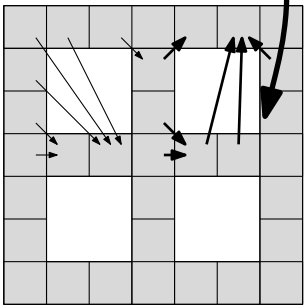




separator flow accumul.



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separator flow directions

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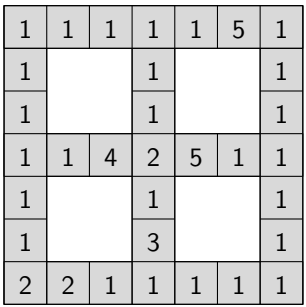
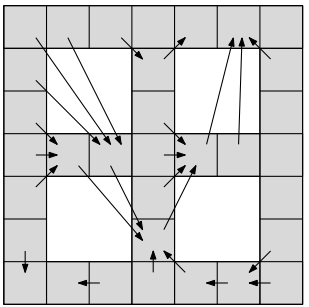


separator flow accumul.



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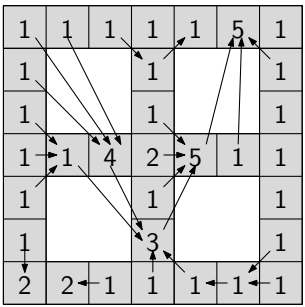

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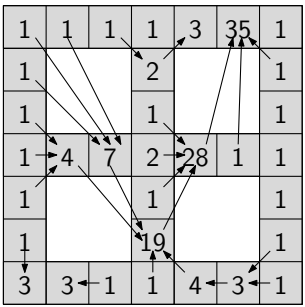

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2. compute flow accumulation of separators

1	1	1	1	3	35	1
1			2			1
1			1			1
1	4	7	2	28	1	1
1			1			1
1			19			1
3	3	1	1	4	3	1

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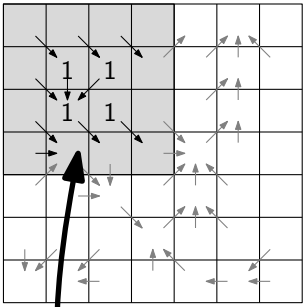

1	1	1	1	3	35	1
1			2			1
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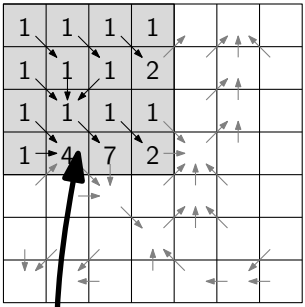

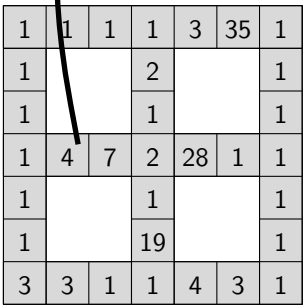

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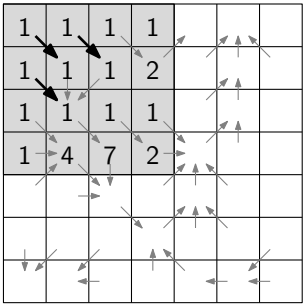

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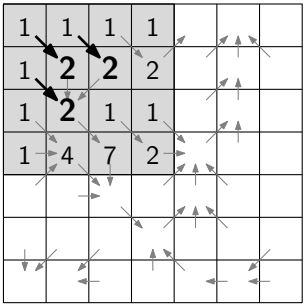

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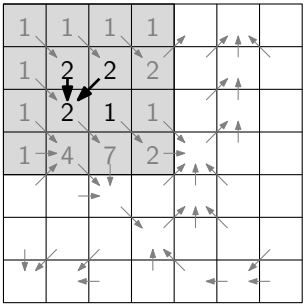

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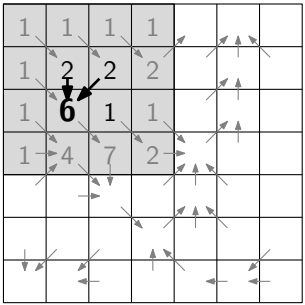

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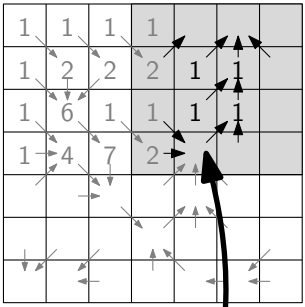

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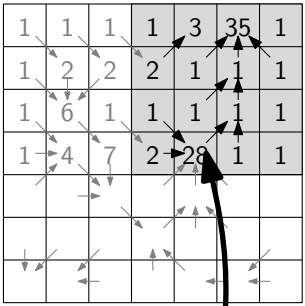

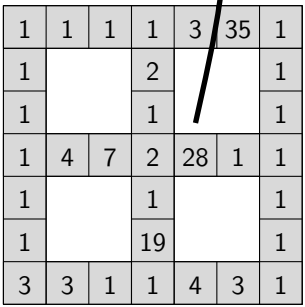

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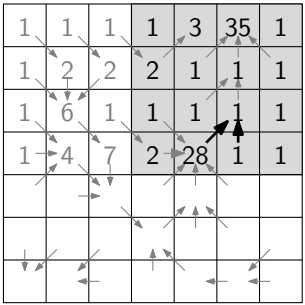

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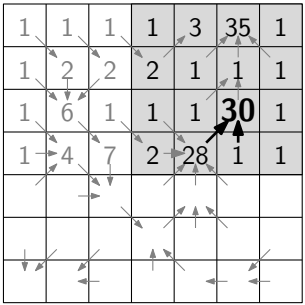

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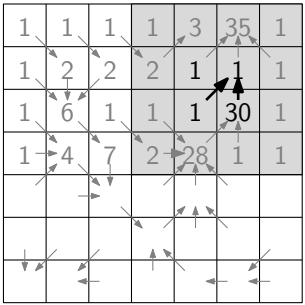

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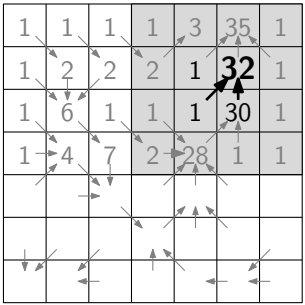

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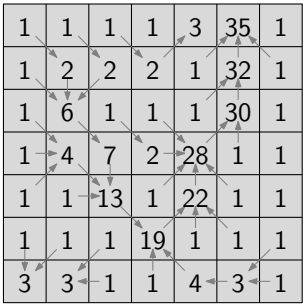

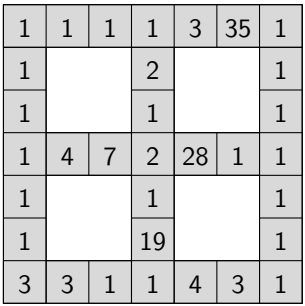

Cache-aware separator-based algorithm

1. move flow from interior of subgrids to separators and compute flow connections between separators	1. $\Theta(N/M)$ subgrids $\times$ $\Theta(M/B + \sqrt{M}) =$ $\Theta(M/B) =$ $\Theta(\mathrm{scan}(N))$ I/O's	1. 1 byte of I/O per cell
2. compute flow accumulation of separators	2. linear-time algo, input $\Theta(N/M) = \Theta(\mathrm{scan}(N))$	2. no I/O in practice
3. move flow from separators into subgrids	3. $\Theta(\mathrm{scan}(N))$ I/O's (like phase 1)	3. 9 bytes of I/O per cell
		Total: 10 bytes per cell if grid stored in Z-order
		Total: 20 to 60 bytes if grid stored row by row (for $1/4 \leq M/B^2 \leq 4$)

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1			2			1
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Results on flow accumulation

algorithm	file order	worst case	'realistic'	bytes per cell	time (mins)
row-by-row scan	row by row	$O(N)$	$O(N/\sqrt{B})$	tenthousands	111
Z-order scan	row by row	$O(N)$	$O(\mathrm{scan}(N))^*$	thousands	
Z-order scan	Z-order	$O(N/\sqrt{B})$	$O(\mathrm{scan}(N))$	hundreds	41
$-aware separ.	row by row	$O(\mathrm{scan}(N))^*$		20 to 60	39
$-aware separ.	Z-order	$O(\mathrm{scan}(N))$		10	run this!
$-obliv. separ.	row by row	$O(\mathrm{scan}(N))^*$		>100	
$-obliv. separ.	Z-order	$O(\mathrm{scan}(N))^*$		>100	118
time-fwd proc.	any	$O(\mathrm{sort}(N))$		70 to 300	sev. hundred
row $\leftrightarrow$ Z-order		$O(\mathrm{scan}(N))^*$		16	88

I/O-volume: $N = 2^{32}$, $M = 1$ GB, $B = 16$ to 64KB

time: 3 GHz Pentium, one disk for data + scratch, $N = 3.5 \cdot 10^9$ (Neuse), $M = 1$ GB

*) needs tall cache

Further research

- separator-based flooding works well too (146 minutes), but do not know how to do controlled/partial flooding
- grid techniques also seem applicable to Pfafstetter watershed labelling
- how about flow routing?
- how about multiple flow direction models? (time-forward processing can do that, can we?)
- does the confluence assumption make sense?
  - is γ indeed constant for realistic terrains under increasing resolution?
  - what are typical values for γ?
  - can we come up with an algorithm to compute γ for a given terrain?
  - can we refine the analysis of the scanning algorithms to explain their good performance?